Trade Policy Analysis in Brazil: Assessing Welfare Impacts with Revised Armington Elasticities*

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Abstract

Evaluating the impacts of trade policy through simulation can offer valuable insights, especially for developing countries still confronting significant trade barriers. However, the credibility of these insights might be compromised if the simulations depend on parameters derived from developed economies or are constrained by ad-hoc assumptions within the literature. To quantify these implications, we focus on Brazil as a case study, utilizing a Melitz-style model that facilitates sector-specific estimation of both an upper-tier "home-foreign" macroelasticity and a lower-tier "foreign-foreign" microelasticity. While 66-77% of microelasticities surpass macroelasticities, the precision falls short in confirming the "rule of two" — the assumption that the microelasticity is twice the macroelasticity. In a simulated trade liberalization scenario, our findings show that welfare changes fall below those of models adhering to this rule, yet exceed those of approaches overlooking differentiation between the two elasticities. Furthermore, incorporating imperfect factor mobility results in more moderated predictions for welfare outcomes.

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1 Introduction

Armington (or trade) elasticities are the cornerstones of any computable general equilibrium model (CGE) of international economics. In the context of CGE models, these parameters reflect the extent to which imported goods are substitutable between origins and dictate how consumers' demand for goods produced in different countries respond to relative-price changes. Even more interestingly, in several specifications, trade elasticities govern not only consumer behavior but also directly affect firm-side variables (such as markup-prices or market entry). As noted by Dixon and Jorgenson (2012), "*it is no exaggeration to say that (...) [the trade elasticity] is the most important parameter in modern trade theory*".

Several studies indeed reveal that the predictions of CGE simulations are highly sensible to the values assumed for Armington elasticities. For instance, by inputting different commonly reported estimates for trade elasticities to a GTAPinGAMS model, Mc Daniel and Balistreri (2003) find that a unilateral liberalization of trade in Colombia can lead to either gains or losses in welfare, depending on the values adopted for the parameter. More recently, Bekkers et al. (2020) show that variations of 20% in upper- and lower-tier elasticities assumed by GTM and GDYN models can lead to changes of up to 6% in the baseline export values predicted by their simulations. Robustness checks conducted by Schürenberg-Frosch (2015) on different types of CGE models provide further evidence on the relevance of the trade elasticity for the results of these type of exercises.

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Despite the importance of this parameter for the reliability of CGE simulations, few studies adopt strategies to tailor the magnitudes of inputted trade elasticities to the specificities of their analyses. Instead, most exercises still employ elasticities "*that are based on 'guestimation' or on estimates picked from the literature*" (Welsch, 2008). In addition, in many cases, it has also become common practice to calibrate the values of these parameters by means of exogenous simplistic methodologies. For instance, in models with nested preferences, there is a widespread use of the so called "rule of two", where the values of upper-tier (or macro) elasticities are simply tied to half of those assumed for the corresponding lower-tier (or micro) elasticities (Hillberry and Hummels, 2013). Analyses that are more concerned with embedding estimated values for their key parameters have only recently become more popular, with the emergence of the so called "New Quantitative Trade Models".[1]

In either case, plugging-in non-estimated values of trade elasticities to these simulations is risky, since external parameters usually cannot reflect functionalities of the adopted models nor specificities of the countries and sectors in which they focus. For instance, Wunderlich and Kohler (2018) use the CAPRI model to grasp the impact of a free trade agreement between Switzerland and the EU and find that, depending on the sector, the differences between the estimated changes in Swiss imports can vary from as low as -17.7% to as high +10.0% when employing empirically estimated magnitudes for the elasticities instead of typical values picked from the literature. In a similar exercise, Olekseyuk and Schürenberg-Frosch (2016) calculate the outcomes of a trade liberalization policy in the Czech Republic using the GTAP model with self-estimated trade elasticities and show that these are significantly different from the predictions of a baseline setup, that employs pre-defined synthetic values for these parameters. According to the study, differences in predicted Czech imports could range from -84% to +30% between the two setups, depending on the sector. In the case of CGE simulations that focus on emerging economies, it is also likely that some of their published results suffer from these types of inconsistencies. Although studies that explore these discrepancies are still scarce in such countries, the data required for suitable estimations of Armington elasticities is frequently unavailable in these regions, which has made the use of ad hoc estimates pervasive in most analyses.

In this work, we follow the model specification proposed by Feenstra et al. (2018) (hereafter FLOR) to estimate Armington elasticities for 25 sectors of the Brazilian economy. The model's general structure is that of the seminal works of Melitz (2003) and Chaney (2008), in which consumers have typical Armington-like preferences and heterogenous firms compete monopolistically in markets separated by iceberg and fixed-entry costs. In their version of the model, FLOR introduce a system of nested CES preferences that allows macro and micro trade elasticities to differ (with the former controlling substitution between domestic and imported products and the latter ruling substitution between goods from different foreign suppliers). To construct our general equilibrium model, which is used in numerical simulations of trade policy,we further enhance this specification so as to accommodate intermediate goods, enable import tariff shocks and account for imperfect mobility of factors of production.

Our work relates to other studies aimed at producing structural estimates for quantitative trade models parameters, such as Broda and Weinstein (2006), Soderbery (2015, 2018) and Feenstra and Weinstein (2017). We identify two major contributions of our analysis to this strand of the literature. First, we produce structural estimates of both the micro and the macro Armington elasticities for different sectors of the Brazilian economy. To the best of our knowledge, this is the first study to jointly estimate these parameters for this economy at the sectoral level. Second, by allowing for import tariff shocks and accounting for imperfect mobility of factors of production, we make the FLOR model more suitable numerical simulations for answering real policy questions, especially those related to trade liberalization in emerging countries. Several of these markets are still significantly closed, with Brazilian average import taxes being, for instance, 75% higher than those of economies of similar income.[2] Although most models predict long run welfare gains from the reduction of such barriers, most of these simulations rely on hypotheses of sustained reallocation of factors, which have not historically been observed after liberalization movements in some of those regions (Dix-Carneiro and Kovak, 2017).

The median and the cross-sector average of our estimates for the microelasticities are respectively 3.26 and

[1] See Bekkers (2017) for a discussion on this literature.

[2] In 2019, the average import tariff was of 7.66% in upper-middle-income countries and of 13.43% in Brazil. Data source: https://data.worldbank.org/indicator/TM.TAX.MRCH.SM.AR.ZS.

3.21. The most recent estimates of sectoral trade elasticities for Brazil were those of Barroso (2010), who reports a cross-sector average for the microelasticities of 7.13. Soderbery (2018) finds a median of 2.85 and a mean of 3.37 for Brazil's microelasticities using a database at the 4 digit level of the Harmonized System (HS). For the US, Broda and Weinstein (2006) reports a median of 3.1 and a mean of 12.6 for σ's estimated at the ten-digit Harmonized Tariff System (HTS), between 1990 and 2001.

Estimates for the macroelasticities are respectively of 2.24 and 3.92 under the two-Stage Least Squares (2SLS) and the two-step generalized method of moments (2GMM) specifications. These figures are higher than the sector-average reported by Tourinho et al. (2015), of 1.34, for similar sectors of the Brazilian economy. Although between 65.8% and 77% of our σ's lie above their corresponding ω's, we can reject the null hypothesis that $\omega \leq \sigma$ for no more than 18.8% of the products. The "rule of two" is rejected for about 12% to 26.1% of the goods.

In a final exercise, we explored different parameter settings of the model to simulate the impacts of a complete elimination of tariffs in Brazil. The changes in welfare depend highly on the assumed relationship between σ and ω. For $\sigma = \omega$, the impact on welfare is negative, whereas for $\omega = \sigma/2$, the impacts are positive. Using our estimates of ω and σ, we also find a positive impact on welfare, albeit smaller than with $\omega = \sigma/2$. When imposing imperfect mobility for production factors, the results tend to be smaller. Sectoral outcomes are similarly highly sensitive to these parameters. For instance, the impact on the sectoral production of Textiles, Wearing Apparel and Leather Products is 6.5 times greater if we assume $\sigma = \omega$ instead of using the estimates found in this study.

The paper proceeds as follows. In section 2, we describe the FLOR model of international trade and our minor extensions to its structure, namely, the introduction of intermediate goods, import tariffs, and imperfect mobility of factors of production. In Section 3, we discuss FLOR's approach for estimating the micro and the macroelasticities. We briefly show how endogeneities arise from a mismatch between observed and theoretical prices and how FLOR exploit a panel data model along with assumptions on the error terms of demand and supply curves to mitigate these biases. In Section 4, we detail how we build our database. In Section 5, we present our estimates for the micro and macroelasticities and simulate the results of a hypothetical policy of tariff liberalization in Brazil using both our estimates and benchmark values for σ and ω. We conclude in Section 6. Derivations and specific details about the simulations are relegated to the Appendices.

2 The model

Our model considers a world economy with J regions, G sectors, and F factors of production. The indexes i and j are used to identify regions, the indexes g and k indicate sectors and the index f refers to factors of production. The demand for goods is made up of intermediate consumption, which depicts input-output linkages between sectors, and final consumption. The total demand of each region and each product is an aggregation between goods of different regions according to the nested Armington structure presented in FLOR. Additionally, the model considers that each sector is composed of heterogeneous firms in terms of productivity and that there are fixed entry costs and fixed costs for trade between two regions based on Melitz (2003). Each firm produces a variety under monopolistic competition. Finally, as Bekkers and Francois (2018), we consider imperfect mobility of factors of production.

2.1 Demand for varieties

Given a total expenditure on goods of sector g by region j, V_j^g, the allocation among the different regions follows a nested Armington structure in line with FLOR. At the first level, the total expenditure is divided between expenditure on domestic and imported goods. CES preferences are assumed with elasticity of substitution ω_j^g. This parameter is called the macroelasticity and governs the rate of substitution between domestic and imported goods for relative price changes. Given this system of preferences, the expenditure on domestic and imported goods are

$$V_{jj}^g = \beta_{jj}^g \left(\frac{P_{jj}^g}{P_j^g} \right)^{1-\omega_j^g} V_j^g, \tag{1}$$

$$V_{F_j}^g = \beta_{F_j}^g \left(\frac{P_{F_j}^g}{P_j^g} \right)^{1-\omega_j^g} V_j^g, \tag{2}$$

where V_{jj}^g and $V_{F_j}^g$ are, respectively, the expenditures on domestic and imported goods of sector g by region j, β_{jj}^g and $\beta_{F_j}^g$ are preference parameters, P_{jj}^g and P_{F_j} are the price indexes of the composite goods of domestic and imported varieties, and P_j^g is the price index of the composite good that combines domestic and imported varieties.

At the second level, the expenditure on imported varieties is an aggregation of the expenditure in goods from all possible sources of imports. We once again consider CES preferences, but now with an elasticity of substitution σ_j^g. This parameter, known as the microelasticity, governs the rate of substitution between the different sources of imports. The expenditure on goods from each source $i \neq j$ is therefore given by

$$V_{ij}^g = \kappa_{ij}^g \left(\frac{P_{ij}^g}{P_{F_j}^g} \right)^{1-\sigma_j^g} V_{F_j}^g, \tag{3}$$

where V_{ij}^g is the expenditure on goods from sector g exported by region i to region j, κ_{ij}^g is a preference parameter, and P_{ij}^g is the price index of the composite good of varieties of sector g exported by region i to region j.

Lastly, the representative consumer has also CES preferences over varieties supplied by each region and sector. As in FLOR, the same elasticity of substitution for the second level, σ_j^g, is assumed. There is a mass of varieties supplied to region j by region i, N_{ij}^g, which is endogenously defined. Each variety is indexed by φ. Given an expenditure V_{ij}, the demand for each variety,[3] $c_{ij}^g(\varphi)$, is

$$c_{ij}^g(\varphi) = \left[\frac{p_{ij}^g(\varphi)}{P_{ij}^g} \right]^{-\sigma_j^g} \frac{V_{ij}^g}{P_{ij}^g}, \tag{4}$$

where $c_{ij}^g(\varphi)$ is the demand for the variety φ from sector g supplied by region i to market j, $p_{ij}^g(\varphi)$ is the tariff-inclusive price paid by consumer. Equation (4) is important because it defines the demand curve faced by a firm producing a variety φ to be sold to destination j.

The price index of the composite goods of domestic and imported varieties is

$$P_j^g = \left[\beta_{jj}^g (P_{jj}^g)^{1-\omega_j^g} + \beta_{F_j}^g (P_{F_j}^g)^{1-\omega_j^g} \right]^{\frac{1}{1-\omega_j^g}}, \tag{5}$$

the price index of the composite good of imported varieties is

$$P_{F_j}^g = \left[\sum_{i=1, i \neq j}^{J} \kappa_{ij}^g (P_{ij}^g)^{1-\sigma_j^g} \right]^{\frac{1}{1-\sigma_j^g}}, \tag{6}$$

and the bilateral price index of the composite good of sector g varieties sold by region i to region j is

$$\begin{aligned} P_{ij}^g &= \left\{ \int_{N_{ij}^g} \left[p_{ij}^g(\varphi) \right]^{1-\sigma_j^g} d\varphi \right\}^{\frac{1}{1-\sigma_j^g}} \\ &= \left\{ N_{ij}^g \left[p_{ij}^g(\tilde{\varphi}_{ij}^g) \right]^{1-\sigma_j^g} \right\}^{\frac{1}{1-\sigma_j^g}}, \end{aligned} \tag{7}$$

where N_{ij}^g is the mass of firms from sector g in region i that export to region j, $p_{ij}^g(\varphi)$ is the consumer price of the good exported by a firm with productivity φ and $p_{ij}^g(\tilde{\varphi}_{ij}^g)$ is the consumer price of the average variety conditional on entry into destination j. The consumer price and the productivity of the average firm are defined in the next subsections.

[3] Note that the quantity consumed is net of iceberg costs. In the presence of iceberg costs, $\tau_{ij}^g \geq 1$, the producer exports a quantity $\tau_{ij}^g c_{ij}^g(\varphi)$ so that the consumer effectively consumes the quantity $c_{ij}^g(\varphi)$.

2.2 Production

In each sector, there is a continuum of firms with heterogeneous productivities. As in Melitz (2003), a firm in region i and sector g pays a fixed entry cost, $f_i^{E,g}$, to get a productivity drawn, φ, from a distribution $G_i^g(\varphi)$. Usually, it is assumed that the productivities follow a Pareto distribution, i.e., $G_i^g(\varphi) = 1 - \varphi^{-\gamma_i^g}$, where $\gamma_i^g > 0$. In addition, a firm of region i and sector g has to pay a fixed cost, f_{ij}^g, to export to region j. In order to produce a variety, a firm uses input bundles that combine materials (input-output linkages) and primary factors of production (labor and capital). The fixed costs are also measured in units of input bundles.[4]

The production of input bundles in each sector has a Cobb-Douglas technology. Thus, the unit cost for the input bundles produced in sector g of region i, z_i^g, can be computed as:

$$z_i^g = \frac{\Upsilon_i^g}{A_i^g} \prod_{f=1}^{F} {w_i^{g,f}}^{\gamma_i^{g,f}} \prod_{k=1}^{G} {P_i^k}^{\gamma_i^{g,k}}, \tag{8}$$

where $\sum_{f=1}^{F} \gamma_i^{g,f} + \sum_{k=1}^{G} \gamma_i^{g,k} = 1$, Υ_i^g is a constant term,[5] A_i^g is the sector-specific (common) productivity level in sector g of region i, P_i^k is the price of composite good k used as material in region i, $w_i^{g,f}$ is the price of factor f in region i used in sector g and $\gamma_i^{g,f}$ is a technology parameter that measures the share of the production factor f in the total output value of sector g in region i. Similarly, $\gamma_i^{g,k}$ measures the share of input k in the total output value of sector g in region i.

2.2.1 Profit Maximization

The firm productivity level affects the required quantity of input bundles to produce its variety. A firm with productivity φ in country i exporting to country g needs $y_{ij}^g(\varphi)$ units of input bundles:

$$y_{ij}^g(\varphi) = \frac{\tau_{ij}^g c_{ij}^g(\varphi)}{\varphi} + f_{ij}^g, \tag{9}$$

where $\tau_{ij}^g c_{ij}^g(\varphi)$ is the exported quantity.[6] In the presence of fixed costs, a firm in region i and sector g demands additional f_{ij}^g units of the input bundles in order to export to region j.

A firm in region i and sector g sets a price that maximizes its profit in supplying goods to region j. The firm problem is:

$$\pi_{ij}^g(\varphi) = \max_{p_{ij}^g(\varphi)\geq 0} \left\{ \frac{p_{ij}^g(\varphi)}{1+t_{ij}^g} c_{ij}^g(\varphi) - \frac{z_i^g}{\varphi} \tau_{ij}^g c_{ij}^g(\varphi) + z_i^g f_{ij}^g \right\}, \tag{10}$$

subject to Equation (4). Note that the firm revenue, $\frac{p_{ij}^g(\varphi)}{1+t_{ij}^g} c_{ij}^g$, is net-of-tariffs, where t_{ij}^g is the tariff applied by country j to varieties of sector g from region i. Solving this problem, the optimal price of the firm with productivity φ is

$$\frac{p_{ij}^g(\varphi)}{1+t_{ig}^g} = \frac{\sigma_j^g}{\sigma_j^g - 1} \frac{\tau_{ij}^g z_i^g}{\varphi}. \tag{11}$$

Note that $\frac{p_{ij}^g(\varphi)}{1+t_{ig}^g}$ defined in the Equation (11) is the optimal producer price, gross of iceberg trade costs.

[4] Caliendo et al. (2023) consider that the fixed costs are measured in units of labor. Akgul et al. (2016) use different setups for the input bundles of their fixed and variable costs, deriving fixed costs solely from the value added by factors of production, where only value added is used in fixed cost bundles. Bekkers and Francois (2018) consider the same input bundles for fixed and variables costs. We stick with the Bekkers and Francois (2018) specification.

[5] $\Upsilon_i^g \equiv \prod_{f=1}^{F} {\gamma_i^{g,f}}^{-\gamma_i^{g,f}} \prod_{k=1}^{G} {\gamma_i^{g,k}}^{-\gamma_i^{g,k}}$.

[6] This quantity is gross of iceberg trade costs.

2.3 Selection and Entry

In each region and sector, there are a mass of firms M_i^g operating and a mass of firms N_{ij}^g ($N_{ij}^g \leq M_i^g$) exporting to a destination j. In the presence of fixed costs, not all firms are profitable in all markets. Only firms with variable profit equal to or greater than fixed export costs will export to a given market. Thus, there is a productivity level φ_{ij}^{g*} in which only firms with the same or higher level will export to a specific market. As presented in Melitz (2003), the zero cutoff productivity condition is given by:

$$z_i^g f_{ij} = \frac{r(\varphi_{ij}^{g*})}{\sigma_j^g}, \tag{12}$$

where $r(\varphi_{ij}^{g*}) = p_{ij}^g(\varphi_{ij}^{g*})c_{ij}^g(\varphi_{ij}^{g*})/(1+t_{ij}^g)$ is the revenue of the marginal firm from region i of sector g that is exporting to region j. This condition defines the mass of operating firms N_{ij}^g on the $i-j$ link. Additionally, as in Balistreri and Rutherford (2013), it is interesting to define this condition in terms of the average (CES-weighted) productivity level conditional on entry, $\tilde{\varphi}_{ij}^g$:

$$\tilde{\varphi}_{ij}^g = \left[\frac{1}{1-G(\varphi_{ij}^{g*})}\int_{\varphi_{ij}^{g*}}^{\infty} \varphi^{\sigma_g^j-1} g(\varphi)d\varphi\right]^{\frac{1}{\sigma_j^g-1}}. \tag{13}$$

Considering the Pareto distribution with parameter γ_i^g, we have that

$$\tilde{\varphi}_{ij}^g = \left[\frac{\gamma_i^g}{\gamma_i^g+1-\sigma_j^g}\right]^{\frac{1}{\sigma_j^g-1}} \varphi_{ij}^{g*}. \tag{14}$$

Applying the optimal price in the revenue formula it is possible to show that

$$\frac{r(\tilde{\varphi}_{ij}^g)}{r(\varphi_{ij}^{g*})} = \left(\frac{\tilde{\varphi}_{ij}^g}{\varphi_{ij}^{g*}}\right)^{\sigma_j^g-1}. \tag{15}$$

Using Equations (14) and (15), the condition of zero cutoff productivity in Equation (12) is rewritten as

$$z_i^g f_{ij} = \frac{p_{ij}^g(\tilde{\varphi}_{ij}^g)c_{ij}^g(\tilde{\varphi}_{ij}^g)}{1+t_{ij}^g}\frac{\gamma_i^g+1-\sigma_j^g}{\gamma_i^g\sigma_j^g}. \tag{16}$$

Given that only firms from region i and sector g with productivity level above φ_{ij}^{g*} will export to region j, we have that $1-G(\varphi_{ij}^{g*}) = N_{ig}^g/M_i^g$. Again, using the Pareto distribution, the average productivity level defined in Equation (14) can be rewritten as

$$\tilde{\varphi}_{ij}^g = \left[\frac{\gamma_i^g}{\gamma_i^g+1-\sigma_j^g}\right]^{\frac{1}{\sigma_j^g-1}} \left(\frac{N_{ij}^g}{M_i^g}\right)^{-\frac{1}{\gamma_i^g}}. \tag{17}$$

Finally, the free entry condition is used to determine the mass of operating firms, M_i^g. According to Melitz (2003) and also detailed in Balistreri and Rutherford (2013), an entering firm in region i and sector g has to pay a fixed cost $z_i^g f_i^{E,g}$ to operate. This payment is made only once. After the firm discovers its productivity, it can decide to produce or immediately exit and not produce. A firm that decides to produce is subject to a bad shock with δ probability in each period that will force its exit from the market. In steady-state, the mass of firms that exit the market as a result of this shock is δM_i^g and the total payment is $z_i^g f_i^{E,g}\delta M_i^g$. Individually for a firm, the expenditure per period to cover fixed entry costs is equal to $z_i^g f_i^{E,g}\delta$. Firms will enter the market until the expected profit is equal to the fixed cost payment.

The average firm in region i and sector g has the following profit at each destination market j:

$$\pi_{ij}^g(\tilde{\varphi}_{ij}^g) = \frac{p_{ij}^g(\tilde{\varphi}_{ij}^g)c_{ij}^g(\tilde{\varphi}_{ij}^g)}{(1+t_{ij}^g)\sigma_j^g} - z_i^g f_{ij}^g. \tag{18}$$

Substituting the fixed costs using Equation (16) and considering the fact that the probability of a firm operating on each link is equal to N_{ij}^g/M_i^g, the free entry condition is given by

$$z_i^g f_i^{E,g}\delta = \sum_{j=1}^{N} \frac{p_{ij}^g(\tilde{\varphi}_{ij}^g)c_{ij}^g(\tilde{\varphi}_{ij}^g)}{1+t_{ij}^g}\frac{\sigma_j^g - 1}{\gamma_i^g\sigma_j^g}\frac{N_{ij}^g}{M_i^g}. \tag{19}$$

2.4 Expenditure

In Subsection 2.1, the demand for varieties was defined given a value of total expenditure by region and sector. Here, we define how this expenditure is generated. There are two main blocks: the representative households that characterize the domestic absorption of each region and the intermediate demand. Domestic absorption represents household consumption, gross investment, and government consumption. Intermediate demand represents the expenditure of performing activities on inputs.

Households In each region there is a representative household that maximizes its utility given Cobb-Douglas preferences and a budget constraint. The household problem is:

$$\begin{aligned} U(C_i^1, ..., C_i^G) = \max_{\{C_n^g\}_{g=1}^G} \prod_{g=1}^{G} C_i^{g\,\alpha_i^g}, \quad \text{where} \sum_{g=1}^{G}\alpha_i^g = 1 \\ \text{s.t.} \sum_{g=1}^{G} P_i^g C_i^g \leq I_i. \end{aligned} \tag{20}$$

The variable C_i^g is the final consumption of the composite good g in region i, α_i^g is a preference parameter, P_i^g is the price index of the composite good g in region i and I_i is the total income in region i. Similar to Caliendo and Parro (2015), the total income[7] in each region is the sum of payments to production factors, total tariff revenue and a trade deficit (or surplus). Formally,

$$I_i = \sum_{g=1}^{G}\sum_{f=1}^{F} w_i^{g,f} L_i^{g,f} + TR_i + D_i, \tag{21}$$

where $w_i^{g,f}$ is the price of the production factor f used in sector g of region j, $L_i^{g,f}$ is the quantity of the factor f used in sector g of region j, TR_i is the total tariff revenue in region i, and D_i is the deficit (surplus) of region i.

Given the household problem, the final expenditure on goods from sector g by region j, $V_i^{g,hh}$, is:

$$V_i^{g,hh} = \alpha_i^g I_i. \tag{22}$$

Intermediate Expenditure The total production of the input bundle in sector k in region i is denoted by Y_i^k. Additionally, considering the Cobb-Douglas technology, the input-output coefficients, $\gamma_i^{k,g}$, and that input bundle production value is equal to $z_i^k Y_i^k$, we have that the intermediate expenditure on goods of sector g in region i, $V_i^{g,int}$, is given by

$$V_i^{g,int} = \sum_{k=1}^{G} \gamma_i^{k,g} z_i^k Y_i^k. \tag{23}$$

Thus, total expenditure is

$$V_i^g = V_i^{g,hh} + V_i^{g,int}. \tag{24}$$

[7]The real income is given by I_i/P_i^{hh}, where $P_i^{hh} = \prod_{g=1}^{G}\left(\frac{P_i^g}{\alpha_i^g}\right)^{\alpha_i^g}$.

2.5 Factor Markets

Our model consider F production factors. Usually, it is assumed a perfect mobility of the production factors between sectors of a specific region. As a result, the price of each factor in each region is uniform. However, Bekkers and Francois (2018) argue that multi-sector models with scale effects are subject to corner solutions and infinitely large effects. Strategies employed in the literature to mitigate this issue include adopting imperfect mobility for production factors, assuming macroelasticities to have lower values than microelasticities and adjustments to the input-output coefficients. The model already allows $\omega < \sigma$, and we also allow the case of imperfect mobility for production factors.[8] We follow Bekkers and Francois (2018) and use a constant elasticity of transformation structure to define the supply of factors in different sectors. In the case of Brazil, the imperfect mobility hypothesis seems to be reasonable given the evidence of Dix-Carneiro and Kovak (2017). More recently, imperfect mobility has also been used in a quantitative dynamic trade model presented in Caliendo et al. (2019).

Given a stock L_i^f of the production factor f in the region i and assuming a constant elasticity of transformation θ_i^f, the factor supply l_i^{gf} in each sector is given by

$$l_i^{gf} = \lambda_i^{gf} \left(\frac{w_i^{gf}}{\bar{w}_i^f} \right)^{\theta_i^f} L_i^f, \tag{25}$$

where λ_i^{gf} is a share parameter, w_i^{gf} is the price of factor f in sector g of region i, $\bar{w}_i^f$ is the price index of factor f in region i. This price index is calculated as

$$\bar{w}_i^f = \sum_{g=1}^{G} \left[\lambda_i^{gf} (w_i^{gf})^{1+\theta_i^f} \right]^{\frac{1}{1+\theta_i^f}}. \tag{26}$$

2.6 Market Clearing

To complete the model, it is necessary to establish the equilibrium conditions for the markets of input bundles and production factors. For the input bundle, there are three sources of uses: use for fixed input costs, use for fixed costs for operation in each bilateral link, and use for variable production. Thus, the equilibrium condition of this market is given by

$$Y_i^g = M_i^g f_i^{E,g} \delta + \sum_{j=1}^{N} N_{ij}^g \left(f_{ij}^g + \frac{\tau_{ij}^g c_{ij}^g (\tilde{\varphi}_{ij}^g)}{\tilde{\varphi}_{ij}^g} \right) \tag{27}$$

For the factor market, the equilibrium price is the one that equalizes the supply and demand of each factor in each sector. Again, considering the Cobb-Douglas production technology, the equilibrium condition is as follows

$$l_i^{gf} = \frac{\gamma_i^{gf} z_i^g Y_i^g}{w_i^{gf}}. \tag{28}$$

3 Estimation

It is often not possible to map the price indexes available in standard databases to those defined by economic theory. In the case of our model, an exact translation of practical price indexes (or unit values) to our CES-like price aggregators would imply gathering data on very specific demand- and supply-side variables, such as taste parameters and quantities of available varieties. Unfortunately, since we cannot access demand and supply curves in practical contexts (but only equilibrium prices and quantities) these variables are seldom observable. To estimate $\hat{\sigma}$ and $\hat{\omega}$ without directly observing our model's price-indexes, we follow FLOR and employ an estimation technique that leverages a series of interconnected moment conditions. These moments

[8] In the numerical simulations, we also adjust the input-output coefficients, following the approach used in Balistreri et al. (2011) and Costinot and Rodríguez-Clare (2014).

arise from theoretical assumptions about the independence of demand and supply curves, which allow us to eliminate the unobservable terms in our model's CES-like price-index.

More formally, consider that panel data is available for expenditures on domestic and imported goods (V_{jj}^{gt}, $V_{F_j}^{gt}$, V_{ij}^{gt}) and for the corresponding unit values of these products (UV_{jj}^{gt}, $UV_{F_j}^{gt}$, UV_{ij}^{gt}).

Combining Equations (1), (2) and (3), we can write the ratio between country j's expenditures on imported goods from origin i and those on domestic goods as:

$$\frac{V_{ij}^g}{V_{jj}^g} = \kappa_{ij}^g \left(\frac{\beta_{F_j}^g}{\beta_{jj}^g}\right) \left(\frac{P_{ij}^g}{P_{F_j}^g}\right)^{1-\sigma_j^g} \left(\frac{P_{F_j}^g}{P_{jj}^g}\right)^{1-\omega_j^g}. \tag{29}$$

Although we may be tempted to promptly estimate Equation (29), we should note that data on the price indexes is unavailable, since these measures are merely theoretical constructs from our model. Instead, we must rely on unit values (UV_{jj}^{gt}, $UV_{F_j}^{gt}$, UV_{ij}^{gt}) to serve as imperfect proxies for P_{jj}^{gt}, $P_{F_j}^{gt}$ and P_{ij}^{gt}.

To understand how these measures relate, we follow FLOR and assume that the unit value UV_{ij}^{gt} generally reflects a consumption-weighted average of prices (of the varieties of good g sold by region i to region j). Using Equations (4) and (11), we can write this weighted average as:[9]

$$\begin{aligned} UV_{ij}^{gt} &= \int_{-\infty}^{\infty} p_{ij}^{gt}(\varphi) \left[\frac{c_{ij}^{gt}(\varphi)}{\int_{\varphi_{ij}^{*gt}}^{\infty} c_{ij}^{gt} dG_i^g(\varphi)}\right] dG_i^g(\varphi) \\ &= \frac{(\gamma_i^g - \sigma_j^g)}{(\gamma_i^g - \sigma_j^g + 1)} p_{ij}^{gt}(\varphi_{ij}^{*gt}). \end{aligned} \tag{30}$$

Taking the ratio between the current and previous unit values of g to eliminate some terms, we get:

$$\begin{aligned} \frac{UV_{ij}^{gt}}{UV_{ij}^{gt-1}} &= \frac{(1+t_{ij}^{gt})\tau_{ij}^{gt}}{(1+t_{ij}^{gt-1})\tau_{ij}^{gt-1}} \left(\frac{z_i^{gt}}{z_i^{gt-1}}\right) \left(\frac{\varphi_{ij}^{*gt-1}}{\varphi_{ij}^{*gt}}\right) \\ &= \frac{(1+t_{ij}^{gt})\tau_{ij}^{gt}}{(1+t_{ij}^{gt-1})\tau_{ij}^{gt-1}} \left(\frac{z_i^{gt}}{z_i^{gt-1}}\right) \left(\frac{N_{ij}^{gt} M_{ij}^{gt-1}}{N_{ij}^{gt-1} M_{ij}^{gt}}\right)^{\frac{1}{\gamma_i^g}}, \end{aligned} \tag{31}$$

where the first and second equalities follow directly from Equations (11) and from the assumption that $1 - G(\varphi*) = N_{ig}^g / M_i^g$, with $\varphi* \sim Pareto(0, \gamma_i^g)$.

Following similar steps, we can use Equations (7), (11) and (17) to achieve an analogous expression for the CES-like price indexes defined in our model:

$$\begin{aligned} \frac{P_{ij}^{gt}}{P_{ij}^{gt-1}} &= \left(\frac{N_{ij}^{gt}}{N_{ij}^{gt-1}}\right)^{\frac{1}{1-\sigma_j^g}} \frac{(1+t_{ij}^{gt})\tau_{ij}^{gt}}{(1+t_{ij}^{gt-1})\tau_{ij}^{gt-1}} \left(\frac{z_i^{gt}}{z_i^{gt-1}}\right) \left(\frac{\tilde{\varphi}_{ij}^{gt-1}}{\tilde{\varphi}_{ij}^{gt}}\right) \\ &= \left(\frac{N_{ij}^{gt}}{N_{ij}^{gt-1}}\right)^{\frac{1}{1-\sigma_j^g}} \frac{(1+t_{ij}^{gt})\tau_{ij}^{gt}}{(1+t_{ij}^{gt-1})\tau_{ij}^{gt-1}} \left(\frac{z_i^{gt}}{z_i^{gt-1}}\right) \left(\frac{N_{ij}^{gt} M_{ij}^{gt-1}}{N_{ij}^{gt-1} M_{ij}^{gt}}\right)^{\frac{1}{\gamma_i^g}}. \end{aligned} \tag{32}$$

Contrasting Equations (31) and (32), we get:

$$\frac{UV_{ij}^{gt}}{UV_{ij}^{gt-1}} = \left(\frac{N_{ij}^{gt}}{N_{ij}^{gt-1}}\right)^{\frac{1}{\sigma_j^g - 1}} \frac{P_{ij}^{gt}}{P_{ij}^{gt-1}}. \tag{33}$$

[9] See details in Appendix A.1.

Equation (33) shows that, although the price index of good g (as defined in our theoretical model) is related to its observable unit value, these two measures are not equivalent. The mismatch between the two variables even survives the elimination of constants that we achieve by taking ratios of current and lagged values of UV_{ij}^{gt} and P_{ij}^{gt}. More importantly, Equation (33) reveals that this difference directly depends on the number of available varieties in each period (N_{ij}^{gt}), a variable that we seldom observe.

Aggregating P_{ij}^{gt} and UV_{ij}^{gt} (for $i \neq j$) into Sato-Vartia indexes, we can establish an analogous relation between a CES-like price index for imported varieties, $P_{F_j}^{gt}$, and its unit value counterpart, $UV_{F_j}^{gt}$. In Appendix A.2 we show that the relation between the one-period changes of these indexes is given by

$$\frac{UV_{F_j}^{gt}}{UV_{F_j}^{gt-1}} = \left(\frac{P_{F_j}^{gt}}{P_{F_j}^{gt-1}}\right)\left(\frac{\kappa_{F_j}^{gt} N_{F_j}^{gt}}{\kappa_{F_j}^{gt-1} N_{F_j}^{gt-1}}\right)^{\frac{1}{\sigma_j^g - 1}} \tag{34}$$

where $\frac{\kappa_{F_j}^{gt}}{\kappa_{F_j}^{gt-1}} = \prod_{i=1,\, i\neq j}^{J}\left(\frac{\kappa_{ij}^{gt}}{\kappa_{ij}^{gt-1}}\right)^{w_{ij}^{gt}}$, $\frac{N_{F_j}^{gt}}{N_{F_j}^{gt-1}} = \prod_{i=1,\, i\neq j}^{J}\left(\frac{N_{ij}^{gt}}{N_{ij}^{gt-1}}\right)^{w_{ij}^{gt}}$ and w_{ij}^{gt} are Sato-Vartia weights.

As in the case of UV_{ij}^{gt} and P_{ij}^{gt}, Equation (34) shows that the one-period changes for the CES-like price indexes also differ from those of the unit values, since the $UV_{F_j}^{gt}$'s are unable to correct for unobservable shocks in taste or changes in product variety.

Using Equations (33) and (34) we can now rewrite Equation (29) in terms of its observable and unobservable terms as

$$\Delta \ln\left(\frac{V_{ij}^{gt}}{V_{jj}^{gt}}\right) = -(\sigma_j^g - 1)\Delta \ln\left(\frac{UV_{ij}^{gt}}{UV_{F_j}^{gt}}\right) + (1-\omega_j^g)\Delta \ln\left(\frac{UV_{F_j}^{gt}}{UV_{jj}^{gt}}\right) + \varepsilon_{ij}^{gt}, \qquad i = 1, ..., J,\ i \neq j,\ t = 2, ..., T, \tag{35}$$

where $\Delta \ln$ denotes log-changes and ε_{ij}^{gt} is the unobservable error term, given by

$$\varepsilon_{ij}^{gt} = \Delta \ln\left(\frac{\kappa_{ij}^{gt}}{\kappa_{F_j}^{gt}}\right) + \Delta \ln\left(\frac{N_{ij}^{gt}}{N_{F_j}^{gt}}\right) + \Delta \ln\left(\frac{\beta_{F_j}^{g}}{\beta_{jj}^{g}}\right) - \frac{(1-\omega_j^g)}{(\sigma_j^g - 1)}\Delta \ln\left(\frac{\kappa_{F_j}^{gt} N_{F_j}^{gt}}{N_{jj}^{gt}}\right). \tag{36}$$

Estimates for the micro- and macroelasticities obtained directly from Equation (35) are expected to be biased, since the changes in unit-values are likely correlated with the variables swept under the error term ε_{ij}^{gt}. As noted by FLOR, for instance, a positive shock in the taste coefficient for goods imported by j from trade partner i (κ_{ij}^{gt}) would directly affect wages in country i, pushing UV_{ij}^{gt} up. Changes in product varieties introduce an additional source of bias, since the ranges N_{ij}^{gt} are endogenously determined by our model while they also affect the relation between true price indexes and unit values (see Equations (33) and (34)).

To mitigate these biases, FLOR propose a methodology that aims at exploiting moment conditions constructed from the error terms of both structural demand equations and reduced form supply curves. In summary, the approach consists of three main steps, which involve separating the micro and macro demand curves, synthesizing their corresponding reduced form supply curves, and estimating their coefficients from moment conditions that arise from combining the error terms of those demand and supply equations. For this last step, FLOR leverages the panel structure of the data and rely on the hypothesized independence between the residuals of these curves across pooled observations. This general procedure is similar for the estimation of both the micro- and the macroelasticity.

We briefly describe each of these steps in the next three subsections.

3.1 Isolating micro and macro structural demand curves

With some algebraic manipulation of our previous expressions, we can construct demand equations that depend uniquely on either the micro or the macroelasticity.

Micro demand curve To isolate the micro demand curve, we sum across Equation (35), for $i \neq j$, and take the difference between this result and the original equation (see Appendix A.3 for details). This operation cancels out the term that depends on ω_j^g and leaves us with

$$\Delta \ln \left(\frac{UV_{ij}^{gt}}{UV_{F_j}^{gt}} \right) = -\frac{1}{(\sigma_j^g - 1)} \Delta \ln \left(\frac{V_{ij}^{gt}}{V_{F_j}^{gt}} \right) + \frac{\varepsilon_{iF}^{gt}}{(\sigma_j^g - 1)}, \tag{37}$$

where

$$\varepsilon_{iF}^{gt} = \Delta \ln \left(\frac{\kappa_{ij}^{gt}}{\kappa_{F_j}^{gt}} \right) + \Delta \ln \left(\frac{N_{ij}^{gt}}{N_{F_j}^{gt}} \right).$$

Macro demand curve To obtain the macro demand curve, we combine Equations (2) and (3) and again sum across trade partners $i \neq j$. Assuming $\omega_j^g = \omega_j$ for a group of products $g = 1...G^s$ within each sector s, we get (see Appendix A.4):

$$\Delta \ln \left(\frac{UV_{F_j}^{gt}}{UV_{jj}^{gt}} \right) = -\frac{1}{(\omega_j - 1)} \Delta \ln \left(\frac{V_{F_j}^{gt}}{V_{jj}^{gt}} \right) + \frac{1}{(\omega_j - 1)} \varepsilon_{F_j}^{gt} \tag{38}$$

with

$$\varepsilon_{F_j}^{gt} = \Delta \ln \left(\frac{\beta_{F_j}^{gt}}{\beta_{jj}^{gt}} \right) + \left(\frac{\omega_j - 1}{\sigma_j^g - 1} \right) \left[\Delta \ln \kappa_{F_j}^{gt} + \Delta \ln \left(\frac{N_{F_j}^{gt}}{N_{jj}^{gt}} \right) \right].$$

The negative signs on the right-hand side (RHS) of Equations (37) and (38) indicate that we should expect a negative correlation between the log-differences of unit values and those of expenditures. We therefore interpret these two expressions as structural demand curves.

3.2 Synthesizing the corresponding reduced form supply curves

To produce consistent estimates for σ_j^g and ω_j^g, FLOR derive auxiliary supply equations, with error terms that are deemed orthogonal to those of their corresponding demand curves.

Micro supply curve To this end, and for the case of the micro supply curve, we start by synthetically drawing a linear projection of $\Delta \ln \left(UV_{ij}^{gt} / UV_{F_j}^{gt} \right)$ onto the error term of the micro demand curve, $\varepsilon_{iF}^{gt}/(\sigma_j^g - 1)$, to get:

$$\Delta \ln \left(\frac{UV_{ij}^{gt}}{UV_{F_j}^{gt}} \right) = \rho_{1j}^g \left(\frac{\varepsilon_{iF}^{gt}}{\sigma_j^g - 1} \right) + \delta_{iF}^{gt}. \tag{39}$$

In other words, for a given destination j and product g, we choose ρ_{1j}^g so as to guarantee that the residuals δ_{iF}^{gt} and ε_{iF}^{gt} are uncorrelated with each other (across all $i \neq j$). Thus, by construction, we have:

$$\sum_t \sum_{i=1,\, i \neq j} \varepsilon_{iF}^{gt} \delta_{iF}^{gt} = 0.$$

This assumption is however insufficient to generate the moments required for a proper estimation of the microelasticities. Note that since we choose a single ρ_{1j}^g for all trade partners of destination j, the synthetic construction of Equation (39) does not guarantee independence between δ_{iF}^{gt} and ε_{iF}^{gt} for each i, but only for all origins taken collectively. The method proposed by FLOR requires a stronger assumption, in which the orthogonality between these residuals hold in expectation for every trade partner of j, such that:

$$E \left[\sum_t \varepsilon_{iF}^{gt} \delta_{iF}^{gt} \right] = 0, \text{ for all } i \neq j. \tag{40}$$

FLOR suggest that this stricter assumption holds by first claiming that Equation (39) can be interpreted as a reduced form (micro) supply curve. This interpretation is then used to produce an additional theoretical argument about the independence between the errors ε_{iF}^{gt} and the residuals of the micro demand curve, δ_{iF}^{gt}.

To show that Equation (39) can indeed be interpreted as a supply curve, FLOR note that a positive shock on ε_{iF}^{gt} (i.e., a relative taste or variety shock on a good imported from i) is expected to generate positive responses both on $\Delta \ln \left(UV_{ij}^{gt} / UV_{F_j}^{gt} \right)$ and $\Delta \ln \left(V_{ij}^{gt} / V_{F_j}^{gt} \right)$. In other words, Equation (39) provides a positive connection, through ε_{iF}^{gt}, between relative unit values and relative expenditures, and we should thus expect $0 < \rho_{1j}^{g} < 1$.

Relying on the supply-side nature of Equation (39) makes the theoretical argument about the independence between ε_{iF}^{gt} and δ_{iF}^{gt} straightforward. Since shifts on the supply and demand curves are usually believed to be caused by unrelated sources, FLOR argue that the unobservable variables comprised in the error terms of these curves must also be influenced by uncorrelated factors. In a subsequent step, FLOR exploit this independence between ε_{iF}^{gt} and δ_{iF}^{gt}, along with the panel structure of their database, to generate the necessary moment conditions for estimating the microelasticity.

Macro supply curve A similar argument can be made to produce analogous moments for the estimation of the macroelasticity. Starting from Equation (38), we can draw a linear projection of $\Delta \ln \left(UV_{F_j}^{gt} / UV_{jj}^{gt} \right)$ on the residual $\varepsilon_{F_j}^{gt} / (\omega_j - 1)$ to get:

$$\Delta \ln \left(\frac{UV_{F_j}^{gt}}{UV_{jj}^{gt}} \right) = \rho_{F_j} \left(\frac{\varepsilon_{F_j}^{gt}}{\omega_j - 1} \right) + \delta_{F_j}^{gt}, \tag{41}$$

for $g = 1...G^s$ (where we again assume $\omega_j^g = \omega_j$ for all goods of a given sector s).

Again, by construction, we know that $\varepsilon_{F_j}^{gt}$ and $\delta_{F_j}^{gt}$ are orthogonal for all goods taken collectively, but not individually for each good. In other words, the synthetical fabrication of the residuals $\delta_{F_j}^{gt}$ does not guarantee that:

$$E \left[\sum_t \varepsilon_{F_j}^{gt} \delta_{F_j}^{gt} \right] = 0, \text{ for all } g \in \{1 \dots G^s\}. \tag{42}$$

As before, we need this latter stronger condition to hold (at this time for producing the estimates for the macroelasticities), so we once more resort to the theoretical argument about the independence of the residuals of demand and supply curves. In this case, we note that positive changes on $\varepsilon_{F_j}^{gt}$ (caused by either variety, taste or preference shocks) will likely generate positive responses both on $\Delta \ln \left(UV_{F_j}^{gt} / UV_{jj}^{gt} \right)$ and $\Delta \ln \left(V_{F_j}^{gt} / V_{jj}^{gt} \right)$, so that $0 < \rho_{F_j} < 1$. Thus, and again following FLOR, we conclude that we can regard Equation (41) as a macro supply curve and that we should therefore expect its residuals $\delta_{F_j}^{gt}$ to be orthogonal to those of the macro demand curve (Equation 38) for each $g \in \{1 \dots G^s\}$.

3.3 Leveraging the panel structure of the data to generate 2SLS and 2GMM estimates for σ_j^g and ω_j

Equations (40) and (42) provide us with $J - 1$ and G^s moment conditions, respectively, that can be used to produce consistent estimates for the micro and macroelasticities.

Estimating the microelasticities For the case of the microelasticity, starting from Equations (37) and (39), we can multiply the error terms ε_{iF}^{gt} and δ_{iF}^{gt} to get:

$$Y_{iF}^{gt} = \theta_1^g X_{1iF}^{gt} + \theta_2^g X_{2iF}^{gt} + u_{iF}^{gt}, \tag{43}$$

where

$$Y_{iF}^{gt} = \left[\Delta \ln\left(\frac{UV_{ij}^{gt}}{UV_{F_j}^{gt}}\right)\right]^2,\ X_{1iF}^{gt} = \left[\Delta \ln\left(\frac{V_{ij}^{gt}}{V_{F_j}^{gt}}\right)\right]^2,\ X_{2iF}^{gt} = \left[\Delta \ln\left(\frac{UV_{ij}^{gt}}{UV_{F_j}^{gt}}\right)\right]\left[\Delta \ln\left(\frac{V_{ij}^{gt}}{V_{F_j}^{gt}}\right)\right],$$

$$\theta_1^g = \frac{\rho_{1j}^g}{(\sigma_j^g - 1)^2(1-\rho_{1j}^g)},\ \theta_2^g = \frac{(2\rho_{1j}^g - 1)}{(\sigma_j^g - 1)(1-\rho_{1j}^g)}$$

and

$$u_{iF}^{gt} = \frac{\varepsilon_{iF}^{gt}\delta_{iF}^{gt}}{(\sigma_j^g - 1)(1-\rho_{1j}^g)}.$$

Notice that Equation (40) guarantees $E\left[\sum_t u_{iF}^{gt}\right] = 0$ for all foreign trade partners. Thus, if we pool Equation (43) over source-countries and sum the resulting groups across time (i.e., over t, for each T_i^g) we get a system of equations with residuals that conveniently sum to zero in expectation for origins $i \neq j$.

A straightforward way to achieve this sort of pooling is to use source-country dummies as exogenous instrumental variables (IVs) and estimate Equation (43) with two-stage least squares (2SLS):

$$\hat{\boldsymbol{\theta}}_{\mathbf{2SLS}}^{\mathbf{g}} = \left[\left(\hat{\mathbf{X}}_{\mathbf{1S}}^{\mathbf{g}}\right)^{\mathbf{T}} \hat{\mathbf{X}}_{\mathbf{1S}}^{\mathbf{g}}\right]^{-\mathbf{1}} \left(\hat{\mathbf{X}}_{\mathbf{1S}}^{\mathbf{g}}\right)^{\mathbf{T}} \mathbf{Y}^{\mathbf{g}}, \tag{44}$$

where

$$\hat{\mathbf{X}}_{\mathbf{1S}}^{\mathbf{g}} = \mathbf{Z}^{\mathbf{g}}\left[(\mathbf{Z}^{\mathbf{g}})^{\mathbf{T}}\mathbf{Z}^{\mathbf{g}}\right]^{-\mathbf{1}}(\mathbf{Z}^{\mathbf{g}})^{\mathbf{T}}\mathbf{X}^{\mathbf{g}}, \tag{45}$$

and $\mathbf{Z}^{\mathbf{g}}$ denote the matrix of source-country indicator variables. In Appendix A.5 we show that the estimates $\hat{\boldsymbol{\theta}}_{\mathbf{2SLS}}^{\mathbf{g}}$ indeed converge asymptotically to the true $\boldsymbol{\theta}^{\mathbf{g}}$ of Equation (43) if $E\left[\sum_t u_{iF}^{gt}\right] = 0$ for all $i \neq j$ (i.e., if the assumption in Equation (40) holds).

In addition, Feenstra (1991) shows that we need an extra assumption on the variances of ε_{iF}^{gt} and δ_{iF}^{gt} to avoid collinearity between the observations of $\hat{\mathbf{X}}_{\mathbf{1S}}^{\mathbf{g}}$, so as to retrieve separable estimates for $\hat{\boldsymbol{\theta}}_{\mathbf{1}}^{\mathbf{g}}$ and $\hat{\boldsymbol{\theta}}_{\mathbf{2}}^{\mathbf{g}}$. More specifically, there should be enough heteroscedasticity between the error terms of the demand and supply curves so as to guarantee that:[10]

$$\frac{\sigma_{\varepsilon_i}^2}{\sigma_{\varepsilon_j}^2} \neq \frac{\sigma_{\delta_i}^2}{\sigma_{\delta_j}^2}, \tag{46}$$

where $\sigma_{\varepsilon_i}^2$ and $\sigma_{\delta_i}^2$ are the variances of ε_{iF}^{gt} and δ_{iF}^{gt}.

With $\hat{\boldsymbol{\theta}}_{\mathbf{1}}^{\mathbf{g}}$ and $\hat{\boldsymbol{\theta}}_{\mathbf{2}}^{\mathbf{g}}$, we can then achieve structural estimates for σ_j^g and ρ_{1j}^g by solving the quadratic equations that arise from the definition of these parameters in Equation (43).

Note that since J is typically greater than 2 (the number of regressors X_{kiF}^{gt} in Equation (43)), we expect our system of equations to be overidentified. We can therefore take advantage of the conventional weighting procedures of the generalized method of moments to improve the efficiency of the 2SLS estimator $\hat{\boldsymbol{\theta}}_{\mathbf{2SLS}}^{\mathbf{g}}$. To this end, we first construct the optimal feasible weighting matrix $\hat{\mathbf{S}}^{\mathbf{g}}$ from the variances of the residuals between $\mathbf{Y}^{\mathbf{g}}$ and $\hat{\mathbf{Y}}_{\mathbf{2SLS}}^{\mathbf{g}}$, so that:

$$\hat{\mathbf{S}}^{\mathbf{g}} = (\mathbf{Z}^{\mathbf{g}})^{\mathbf{T}}\,\mathbf{diag}\left(\hat{\mathbf{u}}_{\mathbf{2SLS}}^{\mathbf{g\_sqd}}\right)\mathbf{Z}^{\mathbf{g}},$$

where $\mathbf{diag}\left[\hat{\mathbf{u}}_{\mathbf{2SLS}}^{\mathbf{g\_sqd}}\right]$ is the diagonal matrix of the squared residuals $\left(\hat{u}_{iF}^{gt}\right)^2$, such that $\hat{u}_{iF}^{gt} = Y_{iF}^{gt} - \hat{\theta}_1^g X_{1iF}^{gt} - \hat{\theta}_2^g X_{2iF}^{gt}$.

In a second step, we weight $\mathbf{Y}^{\mathbf{g}}$ and $\mathbf{X}^{\mathbf{g}}$ by the inverse of $\hat{\mathbf{S}}^{\mathbf{g}}$ (so as to emphasize observations that are more adherent to our model) and re-estimate $\boldsymbol{\theta}^{\mathbf{g}}$ from this new weighted version of Equation (43). Referring back

[10] See Appendix A.6 for details.

to Equation (44), we can therefore write this new 2GMM estimates of $\boldsymbol{\theta}$ in matrix form as:[11]

$$\hat{\boldsymbol{\theta}}^{\mathbf{g}}_{\mathbf{2GMM}} = \left[\left(\mathbf{X^g}\right)^{\mathbf{T}} \mathbf{Z^g} \left(\hat{\mathbf{S}}^{\mathbf{g}}\right)^{\mathbf{-1}} \left(\mathbf{Z^g}\right)^{\mathbf{T}} \mathbf{X^g} \right]^{\mathbf{-1}} \left(\mathbf{X^g}\right)^{\mathbf{T}} \mathbf{Z^g} \left(\hat{\mathbf{S}}^{\mathbf{g}}\right)^{\mathbf{-1}} \left(\mathbf{Z^g}\right)^{\mathbf{T}} \mathbf{Y^g}. \tag{47}$$

Estimating the macroelasticities To produce analogous estimates for the macroelasticity, we repeat these exact same steps and, once again, start by multiplying the error terms of the macro demand and supply curves $\varepsilon^{gt}_{F_j}$ and $\delta^{gt}_{F_j}$ (see Equations (37) and (39)) to get:

$$Y^{gt}_{F_j} = \theta^g_1 X^{gt}_{1F_j} + \theta^g_2 X^{gt}_{2F_j} + u^{gt}_{iF}, \tag{48}$$

where

$$Y^{gt}_{F_j} = \left[\Delta \ln\left(\frac{UV^{gt}_{F_j}}{UV^{gt}_{jj}}\right)\right]^2, \; X^{gt}_{1F_j} = \left[\Delta \ln\left(\frac{V^{gt}_{F_j}}{V^{gt}_{jj}}\right)\right]^2, \; X^{gt}_{2F_j} = \left[\Delta \ln\left(\frac{UV^{gt}_{F_j}}{UV^{gt}_{jj}}\right)\right]\left[\Delta \ln\left(\frac{V^{gt}_{F_j}}{V^{gt}_{jj}}\right)\right],$$

$$\phi_1 = \frac{\rho_{Fj}}{(\omega-1)^2(1-\rho_{Fj})}, \; \phi_2 = \frac{(2\rho_{Fj}-1)}{(\omega-1)(1-\rho_{Fj})}$$

and

$$u^{gt}_{F_j} = \frac{\varepsilon^{gt}_{F_j}\delta^{gt}_{F_j}}{(\omega-1)(1-\rho_{Fj})}.$$

From Equation (42), we know that the residuals $u^{gt}_{F_j}$ should sum to zero in expectation over t within each group of goods. To pool Equation (48) accordingly, we use the same IV-based approach as before, but at this time define our instruments as indicator variables for each $g \in \{1 \dots G^s\}$.

Consistency of $\hat{\boldsymbol{\phi}}^{\mathbf{g}}_{\mathbf{2SLS}}$ and $\hat{\boldsymbol{\phi}}^{\mathbf{g}}_{\mathbf{2GMM}}$ is guaranteed by the assumption on the independence between the residuals of the macro demand and supply curves (Equation (42)) along with an additional condition on the heteroscedasticity of $\varepsilon^{gt}_{F_j}$ and $\delta^{gt}_{F_j}$. Although we do not formally show how convergence occurs in this case, the intuition behind the validity of $\hat{\boldsymbol{\phi}}^{\mathbf{g}}_{\mathbf{2SLS}}$ and $\hat{\boldsymbol{\phi}}^{\mathbf{g}}_{\mathbf{2GMM}}$ under these conditions is exactly the same as that for the estimates of the microelasticity (see Appendices (A.5) and (A.6)).

3.4 Adding an extra set of moment conditions

FLOR point out that, with simulated data, the 2GMM estimates obtained from Equation (48) converge very slowly to the true values of ω. In addition, when carried out with US data, these estimations fail to converge for some sectors.

To tackle these issues, FLOR construct an additional set of moment conditions for the estimation of the macroelasticity. These moments are obtained via the same procedure described in the previous subsections, with steps that again involve isolating a macro demand curve, synthesizing a corresponding reduced-form supply curve, and creating a structural equation with residuals that conveniently cancel out in expectation when pooled properly.

[11] To see that Equation (47) is equivalent to a simple estimation of Equation (44) weighted by the inverse of the estimated residuals $\hat{u}^{gt}_{iF}$, notice that we can rewrite the expression for the 2GMM estimates as:

$$\hat{\boldsymbol{\theta}}^{\mathbf{g}}_{\mathbf{2GMM}} = \left\{ \left(\mathbf{X^g}\right)^{\mathbf{T}} \mathbf{Z^g} \left[\left(\mathbf{Z^g}\right)^{\mathbf{T}} \mathbf{diag}\left(\hat{\mathbf{u}}^{\mathbf{g\_sqd}}_{\mathbf{2SLS}}\right) \mathbf{Z^g} \right]^{\mathbf{-1}} \left(\mathbf{Z^g}\right)^{\mathbf{T}} \mathbf{X^g} \right\}^{\mathbf{-1}} \left(\mathbf{X^g}\right)^{\mathbf{T}} \mathbf{Z^g} \left[\left(\mathbf{Z^g}\right)^{\mathbf{T}} \mathbf{diag}\left(\hat{\mathbf{u}}^{\mathbf{g\_sqd}}_{\mathbf{2SLS}}\right) \mathbf{Z^g} \right]^{\mathbf{-1}} \left(\mathbf{Z^g}\right)^{\mathbf{T}} \mathbf{Y^g} =$$

$$\left\{ \left(\mathbf{X^g}\right)^{\mathbf{T}} \mathbf{Z^g} \left[\left(\mathbf{Z^g}\right)^{\mathbf{T}} \mathbf{diag}\left(\hat{\mathbf{u}}^{\mathbf{g}}_{\mathbf{2SLS}}\right) \mathbf{diag}\left(\hat{\mathbf{u}}^{\mathbf{g}}_{\mathbf{2SLS}}\right) \mathbf{Z^g} \right]^{\mathbf{-1}} \left(\mathbf{Z^g}\right)^{\mathbf{T}} \mathbf{X^g} \right\}^{\mathbf{-1}} \left(\mathbf{X^g}\right)^{\mathbf{T}} \mathbf{Z^g} \left[\left(\mathbf{Z^g}\right)^{\mathbf{T}} \mathbf{diag}\left(\hat{\mathbf{u}}^{\mathbf{g}}_{\mathbf{2SLS}}\right) \mathbf{diag}\left(\hat{\mathbf{u}}^{\mathbf{g}}_{\mathbf{2SLS}}\right) \mathbf{Z^g} \right]^{\mathbf{-1}} \left(\mathbf{Z^g}\right)^{\mathbf{T}} \mathbf{Y^g} =$$

$$\left[\left(\hat{\mathbf{X}}^{\mathbf{g}}_{\mathbf{(1/\hat{u})}}\right)^{\mathbf{T}} \hat{\mathbf{X}}^{\mathbf{g}}_{\mathbf{(1/\hat{u})}} \right]^{\mathbf{-1}} \left(\hat{\mathbf{X}}^{\mathbf{g}}_{\mathbf{(1/\hat{u})}}\right)^{\mathbf{T}} \mathbf{Y}^{\mathbf{g}}_{\mathbf{(1/\hat{u})}}$$

Following FLOR, we start by rewriting Equation (35) with the microelasticities fixed to $\hat{\sigma}_j^g$, taken as given:

$$\Delta \ln\left(\frac{V_{ij}^{gt}}{V_{jj}^{gt}}\right) = -(\omega_j - 1)\Delta \ln\left(\frac{UV_{F_j}^{gt}}{UV_{jj}^{gt}}\right) - (\hat{\sigma}_j^g - 1)\Delta \ln\left(\frac{UV_{ij}^{gt}}{UV_{F_j}^{gt}}\right) + \varepsilon_{ij}^{gt}, \quad i = 1, ..., J,\ i \neq j,\ t = 2, ..., T. \tag{49}$$

where we still consider $\omega_j^g = \omega_j$ for goods of the same sector s and $\hat{\sigma}_j^g$ are those estimated in Subsection 3.3.

Note that this expression is very similar to Equation (38) (in that the macroelasticities ω^j control the inverse relation between relative expenditures and relative unit values), so we can also regard Equation (49) as macro demand curve. Reorganizing the terms to isolate $\Delta \ln\left(UV_{ij}^{gt}/UV_{F_j}^{gt}\right)$, we get:

$$\Delta \ln\left(\frac{UV_{ij}^{gt}}{UV_{F_j}^{gt}}\right) = -\frac{1}{(\hat{\sigma}_j^g - 1)}\Delta \ln\left(\frac{V_{ij}^{gt}}{V_{jj}^{gt}}\right) - \frac{(\omega_j - 1)}{(\hat{\sigma}_j^g - 1)}\Delta \ln\left(\frac{UV_{F_j}^{gt}}{UV_{jj}^{gt}}\right) + \frac{\varepsilon_{ij}^{gt}}{(\hat{\sigma}_j^g - 1)}.$$

Proceeding as before, if we synthetically project $\Delta \ln\left(UV_{ij}^{gt}/UV_{F_j}^{gt}\right)$ onto the two terms in the RHS that are not explicitly linked to import values we get:

$$\Delta \ln\left(\frac{UV_{ij}^{gt}}{UV_{F_j}^{gt}}\right) = \hat{\rho}_{1j}^g \frac{\varepsilon_{ij}^{gt}}{(\hat{\sigma}_j^g - 1)} - \rho_{2j}^g \frac{(\omega_j - 1)}{(\hat{\sigma}_j^g - 1)}\Delta \ln\left(\frac{UV_{F_j}^{gt}}{UV_{jj}^{gt}}\right) + \delta_{ij}^{gt}. \tag{50}$$

where the coefficients $\hat{\rho}_{1j}^g$ are those estimated with Equation (43).

Note that as with our prior projections, the term to the left-hand side (LHS) is expected to be positively influenced by shocks in the terms to the RHS, so we once again expect our synthetic coefficient in the RHS (ρ_{2j}^g) to be greater than 0.[12] From Equation (49), it also directly follows that the terms $\varepsilon_{ij}^{gt}/(\hat{\sigma}_j^g - 1)$ and $-\left[(\omega_j - 1)/(\hat{\sigma}_j^g - 1)\right]\Delta \ln\left(UV_{F_j}^{gt}/UV_{jj}^{gt}\right)$ are both positively correlated with $\Delta \ln\left(V_{ij}^{gt}/V_{jj}^{gt}\right)$. Thus, as before, we can interpret Equation (50) as a macro supply equation, in that it establishes a positive relation between relative unit values $\Delta \ln\left(UV_{ij}^{gt}/UV_{F_j}^{gt}\right)$ and the terms $\varepsilon_{ij}^{gt}/(\hat{\sigma}_j^g - 1)$ and $-\left[(\omega_j - 1)/(\hat{\sigma}_j^g - 1)\right]\Delta \ln\left(UV_{F_j}^{gt}/UV_{jj}^{gt}\right)$, which are direct components of the relative expenditures $\Delta \ln\left(V_{ij}^{gt}/V_{jj}^{gt}\right)$.

Therefore, once again we can apply our previous assumption about the independence of error terms of demand and supply curves, at this time to Equations (49) and (50). As before, if we assume that this assumption holds, the residuals ε_{ij}^{gt} and δ_{ij}^{gt} must be uncorrelated in expectation across all observations for each trade partner $i \neq j$, that is:

$$E\left(\sum_t \varepsilon_{ij}^{gt}\delta_{ij}^{gt}\right) = 0, \text{ for all } i = 1...J,\ i \neq j. \tag{51}$$

Multiplying Equations (49) and (50), we once more end up with a structural equation with error terms u_{ij}^{gt} that conveniently sum to zero if correctly pooled over trade partners $i \neq j$:

$$Y_{iF}^{gt} = \theta_1^g X_{1ij}^{gt} + \theta_2^g X_{2ij}^{gt} + (\omega_j - 1)\left(\theta_3^g X_{3ij}^{gt} + \theta_4^g X_{4ij}^{gt}\right) + (\omega_j - 1)^2 \theta_5^g X_{5ij}^{gt} + u_{ij}^{gt} \tag{52}$$

[12] With regards to the second term, notice that we should expect the relative unilateral unit values to decrease following a positive shock in the multilateral relative unit values $\Delta \ln\left(UV_{F_j}^{gt}/UV_{jj}^{gt}\right)$. As for the components of ε_{ij}^{gt}, the intuition behind the positive correlations between changes in $\Delta \ln\left(UV_{ij}^{gt}/UV_{F_j}^{gt}\right)$ and variety or taste shocks follows from the same arguments presented in the previous subsections.

where

$$Y_{iF}^{gt} = \left[\Delta \ln\left(\frac{UV_{ij}^{gt}}{UV_{F_j}^{gt}}\right)\right]^2, \; X_{1ij}^{gt} = \left[\Delta \ln\left(\frac{V_{ij}^{gt}}{V_{jj}^{gt}}\right)\right]^2, \; X_{2ij}^{gt} = \left[\Delta \ln\left(\frac{UV_{ij}^{gt}}{UV_{F_j}^{gt}}\right)\right]\left[\Delta \ln\left(\frac{V_{ij}^{gt}}{V_{jj}^{gt}}\right)\right],$$

$$X_{3ij}^{gt} = \left[\Delta \ln\left(\frac{UV_{F_j}^{gt}}{UV_{jj}^{gt}}\right)\right]\left[\Delta \ln\left(\frac{UV_{ij}^{gt}}{UV_{F_j}^{gt}}\right)\right], \; X_{4ij}^{gt} = \left[\Delta \ln\left(\frac{UV_{F_j}^{gt}}{UV_{jj}^{gt}}\right)\right]\left[\Delta \ln\left(\frac{V_{ij}^{gt}}{V_{jj}^{gt}}\right)\right], \; X_{5j}^{gt} = \left[\Delta \ln\left(\frac{UV_{F_j}^{gt}}{UV_{jj}^{gt}}\right)\right]^2,$$

$$\theta_1^g = \frac{\hat{\rho}_{1j}^g}{(\hat{\sigma}_j^g - 1)^2(1 - \hat{\rho}_{1j}^g)}, \; \theta_2^g = \frac{(2\hat{\rho}_{1j}^g - 1)}{(\hat{\sigma}_j^g - 1)(1 - \hat{\rho}_{1j}^g)}, \; \theta_3^g = \frac{-(1 + \rho_{2j}^g - 2\hat{\rho}_{1j}^g)}{(\hat{\sigma}_j^g - 1)(1 - \hat{\rho}_{1j}^g)},$$

$$\theta_4^g = \frac{-(\rho_{2j}^g - 2\hat{\rho}_{1j}^g)}{(\hat{\sigma}_j^g - 1)^2(1 - \hat{\rho}_{1j}^g)}, \; \theta_5^g = \frac{-(\rho_{2j}^g - \hat{\rho}_{1j}^g)}{(\hat{\sigma}_j^g - 1)^2(1 - \hat{\rho}_{1j}^g)},$$

and

$$u_{ij}^{gt} = \frac{\varepsilon_{ij}^{gt}\delta_{ij}^{gt}}{(1 - \hat{\rho}_{1j}^g)(\hat{\sigma}_j^g - 1)}.$$

In Appendix A.7 we show that the set of moment conditions that arise from Equation (51) adds extra information to those generated with our previous pair of demand and supply curves (Equations (40) and (42)).

One minor caveat is that, differently from our estimates for the microelasticity, we do not retrieve our structural parameters from solving a system of equations on the estimated coefficients $\hat{\theta}_i^g$. Instead, we obtain $\hat{\omega}_j$ and $\hat{\rho}_{2j}^g$ directly, by first stacking the values for $Y_{F_j/iF}^{gt}$ and $\hat{X}_{1S}^{gt}$ in Equations (52) and (48) and then running a non-linear-least-squares estimation over this complete set of observations.[13]

4 Data

To estimate micro and macroelasticities we need data on domestic consumption and imports for a set of goods of interest. Consumption of goods produced domestically is estimated by subtracting exports from total production for each product. Production data is sourced from two surveys administered by the Brazilian Institute of Geography and Statistics (IBGE): the Annual Industrial Survey - Product (PIA-Produto), and the Municipal Agricultural Production Survey (PAM). The PIA-Produto covers industrial products, while the PAM covers agricultural products.

For the PIA-Produto survey, data from 2005 to 2013 are originally available in the Prodlist 2013 classification, while data from 2014 to 2019 are coded according to newer versions of the Prodlist standard.[14] Using IBGE's correspondence tables,[15] we convert data for all years to Prodlist 2013.[16] In an effort to enhance the reliability of calculated prices, we discard products that had responses from fewer than three firms.

We obtain import and export data for Brazil from the Brazilian Foreign Trade Secretariat (SECEX). The database is detailed by the Mercosur Common Nomenclature (NCM) code,[17] by country of origin (exporter) and by year. We employ CIF import values, FOB export values, and imported/exported quantities in the analysis. Since our model contemplates tariffs, all import values are gross of tariffs. Tariff data for Brazil at the NCM level is obtained from the World Integrated Trade Solution (WITS).

Production and trade data are in different classifications and needed to be made compatible with each other. For agricultural products, we manually mapped PAM products with the correlation between agricultural products and NCM codes available in the Agrostat system of the Ministry of Agriculture of Brazil.[18] For

[13] As in FLOR, we also add a constant term to this specification to control for possible measurement errors in Y_{iF}^{gt}.
[14] More specifically, data from 2014 to 2016 follow Prodlist 2016 and data from 2017 to 2019 adhere to Prodlist 2019.
[15] The correspondence tables are available at: https://concla.ibge.gov.br/classificacoes/correspondencias/produtos.html.
[16] Only products with 1-to-1 or 1-to-n associations between classifications were kept. Associations of type 1-to-n allow products to be aggregated into Prodlist 2013 using their disaggregated codes from Prodlist 2016 and 2019.
[17] In this classification, each product is identified by an 8-digit numerical code.
[18] https://indicadores.agricultura.gov.br/agrostat/index.htm.

industrial products, we used the correspondence table between the NCM and the Prodlist classifications provided by IBGE.[19]

An additional difficulty in matching data is imposed by the periodic revisions of the Harmonized System (HS) of International Trade, which impact the NCM classification. Since correlation tables are available for the Prodlist 2013 and the NCM 2012 classifications, it was necessary to match the NCM 2012 classification with all such revisions from 2005 to 2019. This was done through the use of correlation tables.[20] We kept only products for which these correlation tables allowed either a 1-to-1 or a 1-to-n association between their Prodlist and NCM classifications. The remaining products were discarded, since we could not uncover the exact matches between these two classifications through the available correlation tables. A detailed summary of the data processing and exclusions by sector can be found in Appendix B.

Further, we standardized the units of measurement between production and trade data. Products that could not be directly matched due to differences in their units of measurement were discarded. Following this standardization, we directly calculated unit values from traded or produced values, along with their corresponding quantities. To reconcile the values between the trade database, denominated in USD, and the production database, expressed in Brazilian Reais, we utilized the PTAX exchange rate data from the Brazilian Central Bank.

Table 1 presents the descriptive statistics of the data used in the estimation of the micro and macroelasticities. Our final database covers the period between 2005 and 2019 and is composed of more than 1000 products distributed in 25 sectors.[21,22] The sectors with the highest number of products are Chemicals Products, and Machinery and Equipment (n.e.c.). The average number of suppliers (exporters) per product went up from 19 to 23 between 2005 and 2019. There was also also an increase in the average penetration rate of imports. Data used to produce our estimates covers approximately 43% and 40% of total merchandise imports in Brazil in 2005 and 2019.

To calibrate the model used in the simulation exercise of Subsection 5.2 we use 2014 data from the World Input-Output Database (WIOD). Tariffs for all countries in this database are collected from the Market Access Map System (MAcMap).

[19]The Prodlist and NCM correspondences tables are also available at: https://concla.ibge.gov.br/classificacoes/correspondencias/produtos.html.

[20]The correlation tables are available at: http://www.comexresponde.gov.br/portalmdic/sitio/interna/interna.php?area=5&menu=3361 and https://www.gov.br/produtividade-e-comercio-exterior/pt-br/subjects/camex/trade-strategy/tariffs/common-external-tariff.

[21]The database has data for 1246 products. However, not all products have information for all years. As a result, the total number of products varies between 2005 and 2019.

[22]Manufactured goods are allocated to sectors using the first two digits of the Prodlist code. All agricultural products are aggregated into the same sector.

Table 1: Descriptive statistics.

Sector	2005				2019			
	Number of Goods	Mean Number of Exporters	Mean of Imp. Penetration Ratio	Total Imp. (BRL Million)	Number of Goods	Mean Number of Exporters	Mean of Imp. Penetration Ratio	Total Imp. (BRL Million)
Crop and Animal Production	15	6.40	21.28	3120.22	15	6.40	22.52	10376.02
Oil and Gas	1	11.00	66.03	19072.29	1	7.00	16.95	18992.92
Mining Extraction (Metal Ores)	6	8.00	22.45	1236.81	3	6.33	17.34	2982.25
Mining Extraction (Other)	18	9.72	26.83	504.93	17	13.41	32.29	1365.35
Food Products	67	8.28	14.20	1874.63	76	12.14	16.96	9654.59
Beverages and Tobacco	11	8.55	16.07	1028.06	12	20.92	29.33	5139.95
Textiles	60	17.78	15.25	1965.34	60	19.57	30.27	9065.33
Wearing Apparel	26	20.27	8.04	136.62	29	26.83	20.69	2338.89
Leather and Footwear	7	17.14	15.36	91.43	6	15.83	29.29	53.91
Wood Products	9	13.33	10.62	52.38	11	16.91	25.33	227.04
Paper and Paper Products	20	12.25	9.62	1134.38	20	12.70	5.28	1036.58
Printing and Reproduction of Recorded Media	-	-	-	-	1	42.00	9.43	36.59
Coke and Refined Petroleum Products	7	9.29	36.82	1588.85	10	10.50	28.97	3817.04
Chemical Products	226	16.15	31.40	18338.77	249	20.02	41.63	93337.89
Pharmaceuticals and Medicinal Chemical Products	5	13.80	52.27	130.49	4	14.00	62.36	1310.44
Rubber and Plastic Products	36	27.89	26.61	2689.15	35	35.69	33.08	10576.83
Non-metallic Mineral Products	48	17.35	18.05	1241.75	50	21.38	30.56	4601.75
Basic Metals	37	18.73	15.89	2919.52	47	22.51	25.41	19596.47
Fabricated metal products	68	21.22	17.58	2314.38	66	26.30	31.38	10499.97
Computer and Electronic Products	59	29.73	44.29	7005.14	67	37.94	58.62	49378.70
Electrical Equipment	73	32.56	37.14	4969.93	70	39.73	46.77	23482.31
Machinery and Equipment (n.e.c)	153	19.88	36.86	10491.50	149	23.52	43.06	32711.04
Motor Vehicles, Trailers, and Semi-trailers	18	16.78	16.02	3991.85	20	24.75	27.44	18460.00
Other Transport Equipment	6	9.83	20.44	107.25	6	12.67	13.69	528.85
Other Manufacturing	32	19.84	33.73	929.70	37	23.00	46.96	5762.31
Total	1008	18.83	27.11	86935.38	1061	23.15	36.21	335333.00

Note: Exporter means country of origin. The import penetration rate is calculated as total imports in relation to total consumption. The number of exporters and the penetration rate of imports are calculated for each good and the average is calculated for all the goods in each sector.

5 Results

5.1 2SLS and 2GMM estimates for σ^g and ω

Table 2 shows the 2SLS and 2GMM estimates for the microelasticities. Point estimates reflect the median of $\hat{\sigma}_g$ for 1140 goods, grouped into 25 sectors of the Brazilian economy. It is common practice in the literature to restrict the non-linear optimization problem that arise from estimating $\hat{\sigma}$ to a closed set of parameters that are consistent with theory.[23] Instead, and in line with FLOR, we opt to simply discard 106 goods (from our original set of 1246 products) for which we obtain $\hat{\sigma}_g < 0$ for more than 75% of 1000 bootstrap replications performed initially. As in FLOR, 95% confidence intervals are constructed by means of a nested bootstrap procedure.[24]

2GMM estimates are expected to be more efficient than those of standard two-stage-least-squares, so we consider the former as our preferred results. Table 2 shows that the average of the medians of each sector is 3.21. This figure is lower than the cross-sector average found by Barroso (2010), of 7.13, for similar sectors of the Brazilian economy.[25] Our estimate is also lower than those of benchmark estimates used as default parameters in general-purpose CGE-models. Cross-sector averages of assumed microelasticities are of 4.08 in version 10 of the GTAP database[26] (Aguiar et al., 2019) and of 4.34 in the multi-sector, heterogenous-firm trade model proposed by Caliendo et al. (2023).[27]

The overall median of $\hat{\sigma}_g$ is of 3.26, which is close to that reported by FLOR (3.2) and Broda and Weinstein (2006) (3.1) for US sectors. Figure 1 shows the Kernel densities for the 2SLS and 2GMM estimates along with their lower and upper 95% confidence intervals. Unlike in FLOR, the two densities are quite similar and do not show the efficiency gains that are expected from the 2GMM approach.

Estimates for the macroelasticities are reported in Table 3. Unstacked estimates for ω are obtained through minimization of the error term of Equation (48), while stacked estimates accommodate the restrictions of both Equation (48) and Equation (52). In practice, to obtain the stacked estimates, we pile up the the values for $Y^{gt}_{F_j/iF}$ and $\hat{X}^{gt}_{1S}$ in Equations (48) and (52) and run a non-linear-least-squares estimation over this complete set of observations. Confidence intervals shown in parenthesis are again constructed by means of nested bootstraps. Stacked 2GMM estimates are our preferred results, since Equation (48) is expected to add extra information to the unstacked estimator and since we once again expect the 2GMM estimates to be more efficient than those obtained via unweighted 2SLS.

The last row of Table 3 show estimates for the macroelasticity obtained by pooling data accross all goods in our dataset. Point estimates, in this case, are 2.24 and 3.92 for the stacked 2SLS and 2GMM specifications, respectively. These figures are on average higher than those reported by Tourinho et al. (2015), who estimate a cross-sector average of 1.34 for macroelasticities of similar sectors of the Brazilian economy. Their approach is however quite distinct from ours, in that it employs a time series estimation on a specification that does not account for the microelasticities. Although we ignore previous studies that simultaneously estimate σ and ω for the Brazilian economy, our 2SLS results are somewhat close to those obtained by FLOR for the US, of 2.29. Our estimates are also compatible with the average value of the macroelasticities of the GTAP database, of 2.04 (or exactly half of this for the microelasticities). Bajzik et al. (2020) weigh the different

[23] For instance, Broda and Weinstein (2006) perform a grid-search over $1.05 \leq \sigma \leq 131.5$ whenever their unrestricted optimizations return $\hat{\sigma}_g \leq 1$.

[24] To generate the intervals, we first draw 500 random samples (with replacement) from our original observations (each of which with the size of our initial dataset) and calculate the point estimates of $\hat{\sigma}_g$ for each of these samples. We then draw 100 random samples (again with replacement) from each of these 500 bootstraps (keeping the number of sampled observations equal to those in our original dataset) and use those nested samples to calculate standard deviations for each of the initial 500 point estimates of $\hat{\sigma}_g$. With these standard deviations, we are then able to calculate percentile t confidence intervals (as in MacKinnon (2006)).

[25] Unlike Barroso (2010), we do not find evidence that sectors that are less technology intensive necessarily hold higher trade elasticities.

[26] As discussed in Bekkers and Francois (2018), the elasticities of substitution in the GTAP database are based on estimated tariff elasticities from Hertel et al. (2007). In this case, the elasticity of substitution between imports in the Melitz model, σ, is given by $\sigma^g = \xi_g \epsilon^t_g$, where ξ_g is the degree of granularity and ϵ^t_g is the tariff elasticity. We set the degree of granularity to 2/3.5 (as in Caliendo et al. (2023)).

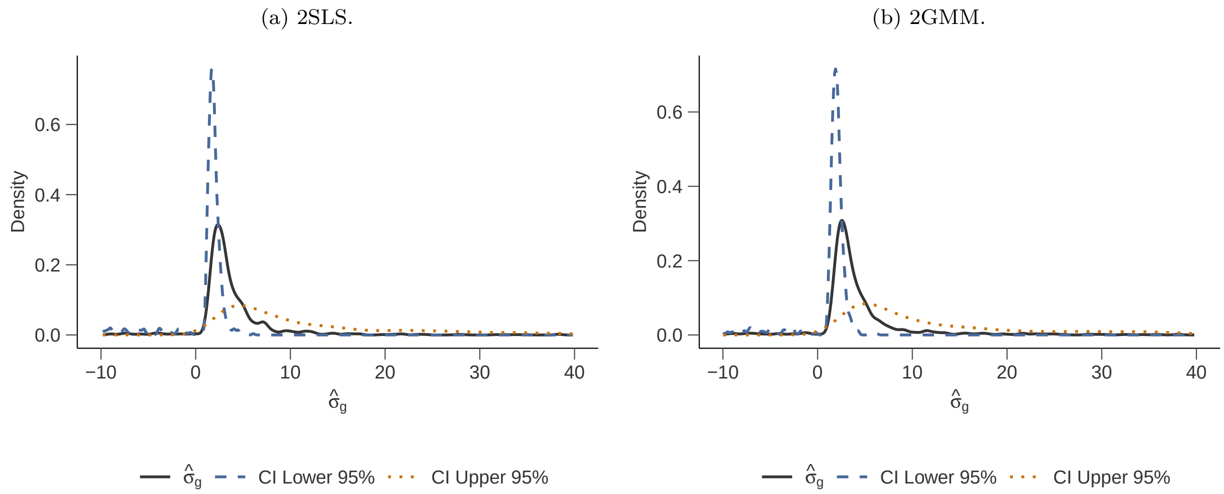
[27] Averages for Aguiar et al. (2019) and Caliendo et al. (2023) are calculated by first mapping these models' sectors into 25 categories that are more compatible with ours.

Table 2: 2SLS and 2GMM estimates of the microelasticities ($\hat{\sigma}_g$).

Sector	Number of goods	2SLS	2GMM
Crop and Animal Production	15	5.20 [3.09; 7.30]	6.51 [4.79; 8.21]
Oil and Gas	1	2.04 [1.56; 4.30]	2.08 [2.11; 3.75]
Mining Extraction (Metal Ores)	6	1.68 [1.31; 2.07]	3.05 [1.93; 4.21]
Mining Extraction (Other)	18	2.67 [2.25; 3.10]	2.45 [1.73; 3.10]
Food Products	79	3.39 [2.82; 4.02]	4.20 [3.71; 4.74]
Beverages and Tobacco	13	2.22 [1.24; 3.08]	3.68 [2.03; 5.23]
Textiles	62	3.73 [3.20; 4.43]	3.62 [3.14; 4.20]
Wearing Apparel	28	4.38 [3.64; 5.04]	5.38 [4.70; 6.03]
Leather and Footwear	8	3.21 [1.95; 4.31]	4.32 [2.75; 5.97]
Wood Products	11	1.28 [0.08; 2.18]	3.72 [1.94; 5.62]
Paper and Paper Products	26	3.96 [3.32; 4.59]	4.01 [3.23; 4.84]
Printing and Reproduction of Recorded Media	1	8.00 [-33.81; 148.16]	-6.47 [-72.29; 145.37]
Coke and Refined Petroleum Products	9	1.75 [0.76; 2.41]	2.50 [1.81; 3.06]
Chemical Products	280	2.59 [2.46; 2.69]	2.68 [2.56; 2.79]
Pharmaceuticals and Medicinal Chemical Products	10	5.52 [2.88; 12.08]	4.05 [1.78; 7.88]
Rubber and Plastic Products	38	3.59 [3.18; 3.92]	3.29 [3.00; 3.58]
Non-metallic Mineral Products	59	2.29 [2.12; 2.46]	2.53 [2.27; 2.77]
Basic Metals	55	3.51 [3.19; 3.86]	3.42 [3.09; 3.76]
Fabricated metal products	71	3.34 [3.00; 3.63]	3.13 [2.80; 3.50]
Computer and Electronic Products		3.70 [2.95; 4.53]	5.69 [4.93; 6.69]
Electrical Equipment	78	3.25 [2.95; 3.57]	3.13 [2.88; 3.35]
Machinery and Equipment (n.e.c)	138	2.74 [2.49; 3.08]	3.13 [2.80; 3.47]
Motor Vehicles, Trailers, and Semi-trailers	17	3.56 [2.24; 5.18]	4.68 [3.23; 6.28]
Other Transport Equipment	6	2.18 [-8.84; 10.42]	2.03 [-6.20; 11.72]
Other Manufacturing	40	3.27 [2.44; 4.26]	3.43 [2.51; 4.51]
All Sectors	1140	3.07 [2.98; 3.16]	3.26 [3.15; 3.35]

Note: Estimates represent the median microelasticity of each sector. Each $\hat{\sigma}_g$ is obtained by solving the system of quadratic equations involving the coefficients θ_i^g of Equation (43). These coefficients are in turn estimated from Equation (43), with the 2SLS and 2GMM procedures described in Subsection 3.3. As in FLOR, the 95% confidence intervals reported in parentheses are obtained from nested bootstrap samples of the observations. All estimates are adjusted by a bootstrap-based bias correction (see Cameron and Trivedi (2005)).

Figure 1: Kernel densities for the overall estimates of the microelasticity.



existing estimates found in the literature (using a list of best practices) and construct a synthetic estimate of 3.2 for the macroelasticity of Brazil, with a confidence interval ranging from 1.3 to 5.0. Caliendo et al. (2023) also adopt values for ω similar to our estimates, which average 2.17 across sectors.[28]

For our stacked specification, the point estimates obtained with 2SLS and with 2GMM are greater than 1 for 92% and 88% of the sectors, respectively. Confidence intervals are however quite large, especially for our 2GMM estimates. Large confidence intervals are also found in FLOR, and stem from the fact that the residuals are not calculated in the same manner across all stacked observations (with some obtained from Equation (48) and others from Equation (52)). The proportion of residuals minimizing each of these two equations is of course uneven between bootstraps, which makes bootstraps estimates become very dissimilar in some cases. This unbalance is more harmful for intervals obtained with the 2GMM method, which makes further use of the uneven residuals to weight the observations of its first step. Lower limits of confidence intervals are greater than one for only 5 out of the 22 sectors for which we obtain consistent 2GMM point estimates. Confidence intervals for the 2SLS estimates are more well-behaved and include the value of unity for 15 out of the 23 sectors with consistent point estimates. In this case, the sectors for which our estimates are not statistically greater than 1 generally feature a very limited number of goods.

Table 4 summarizes three types of statistics that help us compare the sizes of our estimated micro and macroelasticities. To save space, we only compare the 2SLS and 2GMM estimates of σ with the corresponding stacked 2SLS and 2GMM estimates of ω.

In the first column (of each the 2GMM and the 2SLS estimates), we simply report the percentage of goods within each sector for which we have $\hat{\omega} < \hat{\sigma}_g$ (in terms of the point estimates of each of these elasticities). Point estimates for the microelasticity are greater than those of the macroelasticity for 77% and 65.8% of the goods when these are estimated with 2SLS and 2-step-GMM, respectively. These percentages are somewhat homogeneous across sectors (especially for the 2SLS estimates) and are also in line with those of FLOR, who report $\hat{\omega} < \hat{\sigma}$ for 75% of the cases.

The second and third columns (of each the 2GMM and the 2SLS estimates) report the results of statistical tests on two hypotheses about the sizes of σ and ω. As in FLOR, tests are conducted using the nested bootstrap confidence intervals described before. The first test rejects the null hypothesis that $\omega \geq \sigma^g$ for 18.8% and 10.6% of the goods when the elasticities are estimated with the 2SLS and 2GMM methods,

[28]Once again, cross-sector averages for Caliendo et al. (2023) and Aguiar et al. (2019) are obtained after mapping these models' macroelasticities into 25 categories that are more compatible with our sectors. For FLOR, the average is calculated over the study's original 8 sectors.

Table 3: Unstacked and stacked estimates of the macroelasticities ($\hat{\omega}$).

		Unstacked		Stacked	
Sector	Number of goods	2SLS	2GMM	2SLS	2GMM
Crop and Animal Production	15	1.12 [0.26; 2.36]	0.62 [-1.31; 2.30]	1.80 [1.48; 2.13]	2.46 [-1.17; 3.74]
Oil and Gas	1	2.19 [0.76; 6.82]	1.65 [0.83; 3.42]	0.92 [-0.72; 3.12]	1.48 [0.85; 2.80]
Mining Extraction (Metal Ores)	6	15.66 [8.53; 38.03]	8.55 [-9.39; 29.41]	15.69 [9.11; 37.40]	6.47 [-6.95; 20.51]
Mining Extraction (Other)	18	1.07 [0.21; 1.91]	1.61 [0.06; 3.89]	1.41 [0.95; 1.78]	1.37 [-0.36; 3.80]
Food Products	79	4.73 [2.57; 7.15]	2.86 [-7.59; 12.96]	4.16 [2.77; 5.66]	18.05 [11.62; 28.87]
Beverages and Tobacco	13	1.26 [-2.00; 3.91]	4.43 [0.33; 8.50]	2.78 [-1.58; 6.02]	9.15 [4.31; 15.67]
Textiles	62	1.46 [0.85; 2.00]	2.72 [-5.10; 12.46]	2.36 [1.95; 2.90]	1.26 [-5.64; 6.28]
Wearing Apparel	28	3.13 [2.21; 4.35]	2.39 [-3.73; 8.34]	2.95 [2.37; 3.50]	2.69 [-11.79; 24.99]
Leather and Footwear	8	-1.85 [-4.94; 1.54]	-2.65 [-4.90; -0.20]	2.55 [-0.69; 5.20]	2.08 [0.46; 4.27]
Wood Products	11	1.35 [-0.12; 2.81]	-7.33 [-11.23; -4.86]	2.37 [1.37; 3.91]	2.82 [0.74; 4.14]
Paper and Paper Products	26	0.13 [-1.31; 1.48]	2.59 [-0.01; 5.74]	2.17 [1.09; 3.86]	5.39 [2.58; 8.93]
Printing and Reproduction of Recorded Media	1	17.40 [-46.91; 311.81]	13.55 [9.30; 73.63]	16.87 [-57.11; 331.38]	21.52 [9.20; 78.13]
Coke and Refined Petroleum Products	9	-10.39 [-30.15; -3.00]	-0.54 [-6.26; 4.32]	-10.21 [-36.46; -2.86]	-7.06 [-16.26; -3.59]
Chemical Products	280	0.38 [-0.52; 1.04]	-0.76 [-13.50; 3.54]	1.85 [1.31; 2.18]	1.70 [-1.76; 4.67]
Pharmaceuticals and Medicinal Chemical Products	10	0.66 [-2.91; 4.35]	-5.71 [-12.58; -1.16]	2.89 [1.50; 5.80]	0.84 [-2.11; 3.76]
Rubber and Plastic Products	38	-0.28 [-2.81; 0.66]	1.21 [-6.81; 6.33]	2.27 [1.02; 3.96]	1.42 [-5.24; 5.16]
Non-metallic Mineral Products	59	0.95 [-0.90; 2.47]	2.61 [-1.12; 6.38]	1.94 [-0.42; 3.51]	3.51 [-0.22; 7.30]
Basic Metals	55	0.84 [0.30; 1.38]	0.83 [-0.79; 2.25]	2.04 [1.79; 2.29]	2.43 [0.28; 5.26]
Fabricated metal products	71	0.63 [-0.53; 1.83]	4.25 [0.75; 8.04]	2.64 [2.30; 2.83]	7.97 [-4.91; 33.69]
Computer and Electronic Products		0.33 [-0.47; 0.86]	0.45 [-3.53; 2.82]	2.17 [0.54; 3.67]	9.85 [6.95; 16.43]
Electrical Equipment	78	2.50 [1.77; 3.42]	0.32 [-4.60; 4.29]	2.73 [2.35; 3.04]	1.51 [-2.85; 5.24]
Machinery and Equipment (n.e.c)	138	1.67 [1.02; 2.19]	2.84 [0.20; 5.68]	1.88 [1.71; 1.97]	1.62 [-0.06; 3.62]
Motor Vehicles, Trailers, and Semi-trailers	17	-2.93 [-6.21; -0.71]	12.28 [6.22; 16.91]	3.63 [2.22; 5.28]	0.93 [-0.86; 2.49]
Other Transport Equipment	6	1.28 [-1.18; 4.31]	1.02 [-1.92; 4.29]	1.87 [0.75; 3.60]	2.07 [0.44; 3.40]
Other Manufacturing	40	1.72 [0.37; 3.21]	0.64 [-5.15; 6.66]	1.70 [0.77; 2.95]	1.40 [-3.44; 4.95]
All Sectors	1140	1.49 [1.11; 1.64]	2.05 [-1.75; 5.86]	2.24 [2.03; 2.25]	3.92 [-0.64; 7.21]

Note: Unstacked estimates for ω are obtained through minimization of the error term of Equation (48), while stacked estimates minimize the residuals of the system of Equations (48) and Equation (52). These minimizations are achieved by running non-linear-least squares estimations on unweighted (2SLS) and weighted (2GMM) versions of $Y^{gt}_{F_j/iF}$ and $\hat{X}^{gt}_{1S}$ in these systems of equations. Weights for the 2GMM specification are given by the inverse of the residuals between the observed values of $Y^{gt}_{F_j/iF}$ and the estimates $Y^{g\hat{t}}_{F_j/iF}$ obtained with 2SLS. 95% confidence intervals, reported in parentheses, are obtained from nested bootstrap samples of the observations (as in FLOR). All estimates are adjusted by a bootstrap-based bias correction (see Cameron and Trivedi (2005)). The last row represents the estimation results obtained by pooling the products of all sectors and estimating a common ω.

Table 4: Comparing estimated micro (σ^g) and macroelasticities (ω).

	2SLS			2GMM		
		Rejection of H_0:			Rejection of H_0:	
Sector	$\omega < \sigma^g$ (%)	$\omega \geq \sigma^g$ (%)	$2\omega = \sigma^g$ (%)	$\omega < \sigma^g$ (%)	$\omega \geq \sigma^g$ (%)	$2\omega = \sigma^g$ (%)
Crop and Animal Production	92.9	21.4	0.0	78.6	0.0	0.0
Oil and Gas	100.0	0.0	0.0	100.0	100.0	0.0
Mining Extraction (Metal Ores)	0.0	0.0	80.0	33.3	0.0	0.0
Mining Extraction (Other)	88.2	23.5	5.9	94.1	17.6	0.0
Food Products	47.4	6.4	34.6	15.6	2.6	75.3
Beverages and Tobacco	53.8	0.0	0.0	33.3	8.3	41.7
Textiles	83.6	14.8	27.9	88.5	1.6	1.6
Wearing Apparel	80.8	34.6	30.8	96.3	7.4	3.7
Leather and Footwear	75.0	0.0	0.0	71.4	0.0	0.0
Wood Products	36.4	0.0	36.4	60.0	10.0	10.0
Paper and Paper Products	84.6	26.9	11.5	45.8	0.0	29.2
Printing and Reproduction of Recorded Media	0.0	0.0	0.0	0.0	0.0	0.0
Coke and Refined Petroleum Products	88.9	66.7	66.7	88.9	77.8	77.8
Chemical Products	82.1	27.1	37.7	73.2	10.3	1.1
Pharmaceuticals and Medicinal Chemical Products	70.0	0.0	0.0	88.9	0.0	0.0
Rubber and Plastic Products	89.5	15.8	15.8	91.7	13.9	0.0
Non-metallic Mineral Products	91.2	15.8	1.8	44.8	1.7	1.7
Basic Metals	87.3	29.1	34.5	80.8	7.7	0.0
Fabricated metal products	74.6	11.3	31.0	20.0	2.9	4.3
Computer and Electronic Products	76.1	8.5	4.2	37.1	5.7	41.4
Electrical Equipment	84.4	18.2	42.9	93.5	29.9	3.9
Machinery and Equipment (n.e.c)	69.4	20.9	19.4	75.4	16.4	7.5
Motor Vehicles, Trailers, and Semi-trailers	52.9	5.9	11.8	82.4	23.5	5.9
Other Transport Equipment	66.7	0.0	33.3	50.0	33.3	16.7
Other Manufacturing	84.2	13.2	10.5	85.0	10.0	5.0
All Sectors	77.0	18.8	26.1	65.8	10.6	12.0

Note: As in FLOR, test statistics are calculated from nested bootstrapped samples. As indicated in the column headers, statistics refer to 2SLS and 2GMM estimates for σ and to stacked (2SLS or 2GMM) estimates for ω. Tests that involve estimates that are inconsistent with theory are left in blank.

respectively. Thus, depending on the specification, we cannot statistically distinguish between our estimated micro and macroelasticities for about 81.2%% and 89.4% of the goods. Our results for the 2SLS and 2GMM estimates are smaller to those of FLOR, who reject $H_0 : \omega \geq \sigma^g$ for around 35% (2SLS) and 27% (2GMM) of their goods.

The second test assesses the validity of the "rule of two" and rejects the null hypothesis that $2\omega = \sigma^g$ for 26.1% (2SLS) and 12% (2GMM) of the goods. These percentages are broadly in line with those of FLOR, who reject the "rule of two" for about 11% and 20% of their goods with tests employing 2SLS and 2GMM estimates, respectively.

5.2 Application: Tariffs elimination in Brazil

To investigate the importance of the two-tier Armington demand structure, we conduct a set of simulations of a hypothetical reduction to zero of Brazil's import tariffs. Through these simulations, we can assess how the differentiation between microelasticities (σ^g) and macroelasticities (ω^g) can affect the results, and also compare the estimates for Brazil with those reported in the existing literature.[29,30]

[29] In this Subsection, the index g denotes a sector. The index j is dropped since the same elasticitiy of substitution is considered for all regions.

[30] Potential bias may arise from CGE simulations of Free Trade Agreements that impose tariff rate quotas (TRQs), as discussed by Jafari et al. (2021a). While our current model assumes full trade liberalization, bypassing the TRQ-related aggregation bias,

Simulations are conducted following the quantitative trade model outlined in Section 2.[31] In Appendix C, we present additional information regarding the equations comprising the system and the sectors for which either a monopolistically competitive or perfectly competitive structure was assumed. We obtain the numerical solutions using the decomposition method described in Balistreri et al. (2011) and Balistreri and Rutherford (2013). Also, as previously mentioned, multi-sector models with scale effects are subject to the problem of corner solutions and infinitely large effects. To mitigate these issues, the literature commonly uses specific model specifications such as a two-tier Armington structure, imperfect mobility for production factors, and adjustments in input-output coefficients (Bekkers and Francois, 2018). Our simulations allow for these three possibilities, but only the last one is fixed throughout all simulations.[32]

In addition to the elasticities of substitution (σ^g and ω^g), another relevant parameter for the model is the shape parameter of the firm productivity distribution (γ^g). We obtain this parameter from the relation $\zeta^g = \gamma^g/(\sigma^g - 1)$, where ζ^g is the corresponding parameter for the Pareto distribution of firm size. The parameter ζ^g was estimated using the number of employees per firm from the Annual Social Information Report (RAIS) database, following Ahmad and Akgul (2018).[33] For comparison purposes, we simulate the results using our estimates for ω^g and σ^g and benchmark values for these parameters borrowed from the GTAP database and from Caliendo et al. (2023).[34] Regarding the the production factors elasticity of transformation (θ^f), we used the values of 1,[35] and also performed simulations assuming perfect mobility of the production factors.

Our model is calibrated using data primarily sourced from the WIOD, with the exception of tariff data, which was retrieved from the MacMAP database. All simulation data pertain to the year 2014. The database is aggregated to represent five regions (Brazil, China, USA, EU27, and Rest of the World), 24 sectors, and two production factors (capital and labor). To provide context for the exercises, Figure 2 displays the trade-weighted average tariffs imposed by Brazil on a sectoral basis in 2014. There is significant heterogeneity across sectors. It can be observed that the sectors with the highest tariffs are Textiles, Wearing Apparel and Leather Products, Fabricated Metal Products, and Electrical Equipment, while the activities related to agriculture and extractive industries have the lowest tariffs.

The first exercise involves evaluating the welfare impact of tariff liberalization in Brazil for different ratios of the parameters σ^g and ω^g, including the estimated ratio for each sector. We consider the values for σ^g listed in Appendix C. For ω^g, the value is derived from the calculation of the ratio indicated on the abscissa of Figure 3 and the estimated value of σ^g. For example, for a ratio of 1, we compute $\omega^g = \hat{\sigma}^g$, and for a ratio of 2, $\omega^g = \hat{\sigma}/2$. When using our estimates of ω^g, the considered values are also listed in Appendix C.[36]

Figure 3 presents the welfare results for Brazil with the ω^g/σ^g ratio ranging from 1 to 2, as well as the result

more realistic approaches may benefit from the approach proposed by Jafari et al. (2021b) and Jafari et al. (2021a).

[31]Despite not formulating our solution via exact hat algebra, our approach is more consonant with New Quantitative Trade (NQT) models than with Computable General Equilibrium (CGE) models. Our model embodies a simplified user framework, which includes only intermediate and final users, eschewing explicit delineation of investment and government sectors. Furthermore, we deploy a Cobb-Douglas production function, and our Armington elasticity estimates are intrinsically tied to our chosen theoretical model. Contrarily, CGE models are characterized by their granular detail, their integration of parameters from existing literature, and their acceptance of elasticities of substitution between intermediate inputs and production factors that diverge from 1. A more detailed comparison between NQT and CGE models can be accessed in Bekkers (2017).

[32]For details about the adjustment, please refer to Costinot and Rodríguez-Clare (2014). Also, Costinot and Rodríguez-Clare (2014) show that in simulations of a model with constant returns to scale for all sectors, smoothing of input-output coefficients does not drastically alter the results. Appendix C shows the correlation between the sectoral results with and without adjustment of input-output coefficients for a set of simulations in which a structure of perfect competition is assumed for all sectors (multi-sector Armington model).

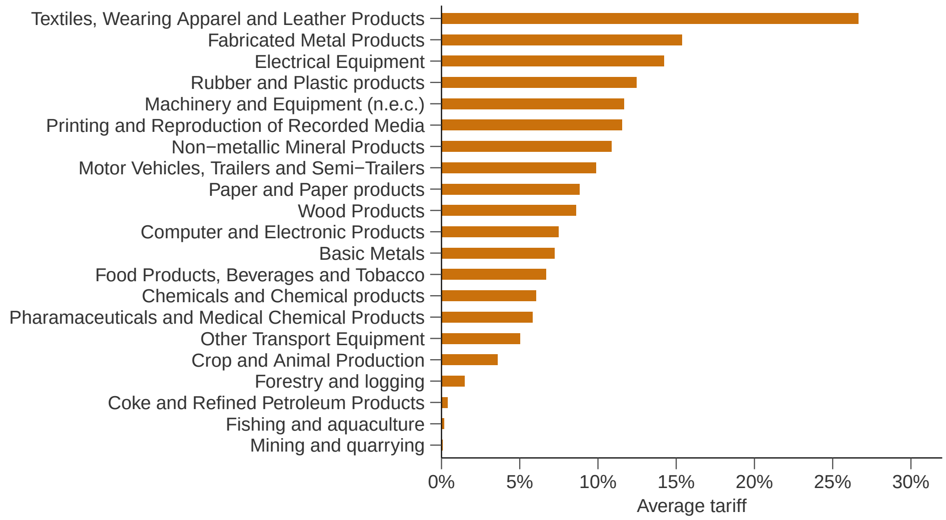
[33]The RAIS database provides a census of the Brazilian formal labor market. We use data from 2019 to estimate ζ^g.

[34]Table 9 shows the Armington elasticities used in each simulation. We use 2GMM estimates for σ^g and ω^g if the lower limit of the confidence interval is greater than one. Otherwise, we use the 2SLS estimate by checking the same rule. If we don't have a valid estimate for a sector, we use the "All Sectors" valid estimates.

[35]This value is arbitrary, but its purpose is to contrast with the results obtained under the assumption of perfect mobility. Also, Bekkers and Francois (2018) use a value of 1 as the lower bound in the sensitivity analysis of this parameter.

[36]Besides using our estimates of σ^g and computing ω^g based on varying ratios of these parameters, an alternative approach could be to use our estimates of ω^g and calculate σ^g for different assumed ratios. However, the solution method doesn't converge for the simulation where $\sigma^g = \hat{\omega}^g$. In our experiments, the algorithm has convergence issues for low elasticity of substitution values. Nevertheless, as the microelasticities are estimated with greater precision, the option suggested in the text may be more suitable.

Figure 2: Average tariff applied by sector - Brazil, 2014



Note: Trade-weighted average tariff calculated using data from the WIOD and the MacMAP databases.

found using our estimates of ω^g and σ^g. The results are shown for simulations that consider both perfect ($\theta^f = \infty$) and imperfect mobility ($\theta^f = 1$) of production factors.[37] It is observed that welfare gain increases with the σ^g/ω^g ratio. This result aligns with the findings of FLOR, who demonstrate that the trade elasticity (ε^g)[38] tends to decrease with the reduction of ω^g, thereby increasing trade gains. The two-tier Armington structure directly impacts welfare variations. This is because, with $1 < \omega^g < \sigma^g$, a decrease in domestic participation in sector g is achieved with a greater decline in the price of imports for that sector (Caliendo et al., 2023). Using our estimates of ω^g, we find a positive welfare impact. A similar result is found by fixing the σ^g/ω^g ratio at, approximately, 1.55 for all sectors.

Figure 3: Analyzing the impact of the ratio between microelasticities (σ^g) and macroelasticities (ω^g) on welfare change resulting from full tariff removal in Brazil.

Note: Welfare is computed as the change in real income (I_i/P_j^{hh}).

[37]The value of 1 for the transformation elasticity (θ^f) is the standard value adopted in Bekkers and Francois (2018).

[38]In the FLOR model, the trade elasticity relevant for welfare analysis is given by: $\varepsilon^g = \frac{\gamma^g(\omega^g-1)}{(\sigma^g-\omega^g)\left(\frac{\gamma^g}{\sigma^g-1}-1\right)+\sigma^g-1}$.

Table 5: Comparison of results for different sets of elasticities.

	Imperfect Mobility ($\theta^f = 1$)			Perfect Mobility ($\theta^f = \infty$)		
Variable	Ours	GTAP	CFRT	Ours	GTAP	CFRT
Welfare	0.11	0.17	0.19	0.17	0.20	0.21
Real Value Added	0.90	1.00	1.04	0.98	1.09	1.12
Total imports	7.60	7.32	6.81	8.41	8.26	7.64
Total exports	13.14	12.67	11.79	14.54	14.29	13.20

Note: The parameter θ_i^f governs factor mobility between sectors. Each simulation was run using a different set of elasticities. 'Ours' means that the elasticities from Tables 2 and 3 were used. In GTAP columns, the GTAP elasticities were considered for all regions. CFRT indicates that the elasticities from Caliendo et al. (2023) were considered for all regions. Welfare is computed as the change in real income (I_i/P_j^{hh}). Real value added is the change in total remuneration of production factors divided by the consumer price index (P_j^{hh}).

Table 5 shows a comparison of outcomes derived from simulations that use our proposed values for σ^g and ω^g with those incorporating elasticities from the GTAP database and from Caliendo et al. (2023). A detailed listing of the elasticity values employed in these simulations is provided in Table 9. Both the GTAP database and Caliendo et al. (2023) adhere to the "rule of two". In contrast, our estimates yield a ratio between σ^g and ω^g that ranges from 1.12 to 3.62.

Firstly, it should be noted that the impact on welfare is lower with our estimates. This result is consistent with the fact that our welfare result could be obtained using a single ratio of approximately 1.55, which is lower than the commonly used "rule of two".

For all simulations, the impact on real value added is positive, ranging from 0.90% to 1.12%. Trade variables are highly impacted, with predicted growth in exports higher than in imports. Although counterintuitive, this result stems from the structure of the model, which assumes constant deficits. Since Brazil recorded a deficit in total trade (of goods plus services) in 2014, the increase in imports must be offset by a relatively greater increase in exports. Despite some differences, it is observed that, at least for the macroeconomic variables, there is no significant discrepancy in the results between the sets of elasticities used in the simulations presented in Table 5.

Regarding the sectoral results for Brazil, Table 6 shows the impacts on the production of each sector for different values of ω^g. For all columns, the results are computed using our estimates for σ^g. We consider three cases. In the first case, we set the value of ω^g equal to our estimate for σ^g. In the second case, we adopt the rule of two and set $\omega^g = \sigma^g/2$. Finally, we use our estimate of ω^g.[39] As in the welfare analysis, the analysis is repeated considering both perfect and imperfect mobility of production factors. It is important to note that ω^g controls the total demand for imports. Thus, when comparing to the scenario where $\omega^g = \sigma^g$, sectoral effects tend to decrease once ω^g is set to a value lower than σ^g. This happens because a lower value for ω^g implies less substitution between domestic and imported goods, resulting in a smaller impact on sectoral production. It is also worth noting that the results obtained in the simulations considering $\omega^g = \sigma^g/2$ are closer to the results obtained with our estimates of σ^g and ω^g than those obtained when we use the same values for these two elasticities.

Looking at specific results, we see that the impacts are quite distinct for the sector of textiles, wearing apparel and leather products, which is a sector with a high initial tariff. In the simulation assuming perfect mobility of production factors, the impact for this sector varies between -54% and -6%, while the impact estimated using the macro elasticity obtained in this study is -8%. As expected, limiting the mobility of production factors reduces these estimates to the range of -20% to -4%.

[39] In Appendix C, we also present the sectoral results for simulations using the elasticities from GTAP and Caliendo et al. (2023).

Table 6: Change in production for different values of the ratio between microelasticities (σ^g) and macroelasticities (ω^g).

	Imperfect Mobility ($\theta^f = 1$)			Perfect Mobility ($\theta^f = \infty$)		
Sector	$\omega^g = \hat{\sigma}^g$	$\omega^g = \hat{\sigma}^g/2$	$\omega^g = \hat{\omega}^g$	$\omega^g = \hat{\sigma}^g$	$\omega^g = \hat{\sigma}^g/2$	$\omega^g = \hat{\omega}^g$
Crop and Animal Production	2.77	1.80	2.00	5.87	2.72	3.09
Forestry and logging	0.80	0.46	0.40	1.03	0.48	0.31
Fishing and aquaculture	1.24	0.61	0.47	2.01	0.77	0.44
Mining and quarrying	4.47	2.29	3.21	14.81	4.30	7.84
Food Products, Beverages and Tobacco	4.03	3.05	3.32	6.73	3.67	4.09
Textiles, Wearing Apparel and Leather Products	-19.97	-3.71	-5.12	-54.31	-5.96	-8.34
Wood Products	2.96	2.69	2.78	5.76	3.84	4.03
Paper and Paper products	3.26	3.30	3.55	6.67	4.35	4.78
Printing and Reproduction of Recorded Media	-1.26	0.57	0.04	-1.92	0.65	-0.12
Coke and Refined Petroleum Products	2.40	1.53	2.24	3.11	1.76	2.57
Chemicals and Chemical products	0.09	1.55	0.90	1.14	1.77	0.96
Pharamaceuticals and Medical Chemical Products	-0.35	0.18	-0.37	-0.29	-0.15	-1.34
Rubber and Plastic products	-4.36	0.42	-1.12	-5.88	0.42	-1.64
Non-metallic Mineral Products	-1.11	1.11	-0.38	-1.44	1.31	-0.74
Basic Metals	4.14	3.89	4.02	8.34	5.05	5.30
Fabricated Metal Products	-6.43	-0.01	-4.03	-11.24	-0.21	-6.81
Computer and Electronic Products	-8.29	-2.69	-0.87	-17.18	-4.04	-1.35
Electrical Equipment	-9.02	-0.26	-6.75	-15.06	-0.52	-11.10
Machinery and Equipment (n.e.c.)	-5.16	0.57	-0.24	-9.30	0.46	-0.72
Motor Vehicles, Trailers and Semi-Trailers	-4.74	0.00	-2.81	-6.35	-0.28	-4.47
Other Transport Equipment	5.01	3.42	3.51	16.27	4.91	5.64
Other Manufacturing	3.35	2.02	2.31	6.08	2.77	3.28
Repair and Installation of Machinery and Equipment	-	-	-	-	-	-
Services	0.12	0.24	0.21	0.14	0.21	0.15

6 Conclusion

CGE simulations of international economics rely heavily on the values that their models assume for Armington elasticities. Despite the importance of these parameters, policy evaluation in emerging countries is still often conducted with CGE models that employ elasticities taken from the literature, which seldom reflect the specific contexts of these regions.

In this work, we estimate Armington elasticities for the Brazilian economy exploring the identification strategy proposed by FLOR. As in FLOR, the model is also flexible enough to allow for sector-specific elasticities and to separate these into an upper-tier parameter (ω), which governs substitution between domestic and imported goods, and a lower-tier coefficient (σ), which rules substitution between goods imported from different trade partners. Using this specification, we employ FLOR's identification strategy to generate consistent estimators for ω and σ. Consistency is achieved through assumptions about the independence and the heteroscedasticity of the residuals of demand and supply equations. Asymptotically, these estimators are able to mitigate potential biases that arise from the mismatch between the model's CES-like price indexes and real-world observable unit values.

Our sector-level estimates for the micro and macroelasticities are somewhat different from those of other recent works on the Brazilian economy. For the microelasticity, our cross-sector average of within-sector medians is of 3.21 for $\hat{\sigma}_g$. The overall macroelasticity estimated by pooling all goods are of 2.24 (2SLS) and 3.92 (2GMM) for $\hat{\omega}$. Barroso (2010) and Tourinho et al. (2015) estimate averages of 7.13 and 1.34, respectively, for the the micro and macroelasticities of similar sectors of the Brazilian economy. Overall, our cross-sector averages for both the micro and macroelasticities are also in line with those of FLOR and with benchmark values of CGE models that differentiate between upper- and lower-tier elasticities.

At least for some sectors, our results fail to corroborate standard assumptions made by CGE models. Depending on the specification, we find that between 65.8% and 77% of the point estimates for σ^g are greater than these for the macroelasticities. In our preferred specification, statistical tests reject $H_0 : \omega \geq \sigma^g$ and $H_0 : 2\omega = \sigma^g$ for 10.6% and 12% of the cases, respectively.

We wrap up the analysis by employing our estimates for ω and σ to simulate the outcomes of an tariff liberalization in Brazil, using an augmented version of FLOR's Melitz-like model, which we enhance to accommodate intermediate goods, import tariffs shocks and imperfect mobility of factors of production. Although our cross-sector averages for $\hat{\sigma}$ and $\hat{\omega}$ apparently indicate that standard calibrations of general-purpose CGE models might properly capture the dynamics of the Brazilian economy, sector level differences between estimated and benchmark coefficients suggest otherwise. For some sectors, the impact of a tariff liberalization on production is considerably different among the sets of parameters considered. In some cases, there is even a change of sign. Our estimates suggest that it would be more appropriate to assume $\omega < \sigma$. However, given the difficulty of obtaining precise estimates and the importance of the ratio between these two parameters for the results, we understand that it would be more appropriate for this ratio to be the subject of a sensitivity analysis in simulations assuming the two-tier Armington structure.

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Appendix

A Derivations

A.1 Unit values

Assuming that unit values UV_{ij}^{gt} reflect a consumption-weighted average of the prices of each variety of good g, we have:

$$UV_{ij}^{gt} = \int_{\varphi_{ij}^{*gt}}^{\infty} p_{ij}^{gt}(\varphi) \left[\frac{c_{ij}^{gt}(\varphi)}{\int_{\varphi_{ij}^{*gt}}^{\infty} c_{ij}^{gt}(\varphi) dG_i^g(\varphi)} \right] dG_i^g(\varphi). \tag{53}$$

Using Equations (4) and (11) we can write $c_{ij}^{gt}(\varphi)$ as

$$\begin{aligned} c_{ij}^{gt}(\varphi) &= \left[\frac{p_{ij}^{gt}(\varphi)}{P_{ij}^{gt}} \right]^{-\sigma_j^g} \frac{V_{ij}^{gt}}{P_{ij}^{gt}} \\ &= \left[\frac{\sigma_j^g (1 + t_{ij}^{gt}) \tau_{ij}^{gt} z_i^{gt}}{(\sigma_j^g - 1) P_{ij}^{gt}} \right]^{-\sigma_j^g} \left(\frac{V_{ij}^{gt}}{P_{ij}^{gt}} \right) (\varphi)^{\sigma_j^g}, \end{aligned} \tag{54}$$

so that Equation (53) becomes:

$$UV_{ij}^{gt} = \int_{\varphi_{ij}^{*gt}}^{\infty} p_{ij}^{gt}(\varphi) \left[\frac{\varphi^{\sigma_j^g}}{\int_{\varphi_{ij}^{*gt}}^{\infty} \varphi^{\sigma_j^g} dG_i^g(\varphi)} \right] dG_i^g(\varphi). \tag{55}$$

Assuming a Pareto distribution for φ (i.e. $G_i^g(\varphi) = 1 - \varphi^{-\gamma_i^g}$) and imposing $\gamma_i^g > \sigma_j^g$, we have:

$$\begin{aligned} UV_{ij}^{gt} &= \int_{\varphi_{ij}^{*gt}}^{\infty} p_{ij}^{gt}(\varphi) \left[\frac{\varphi^{\sigma_j^g}}{\gamma_i^g \int_{\varphi_{ij}^{*gt}}^{\infty} \varphi^{\sigma_j^g - \gamma_i^g - 1} d\varphi} \right] dG_i^g(\varphi) \\ &= \int_{\varphi_{ij}^{*gt}}^{\infty} p_{ij}^{gt}(\varphi) \left[\frac{\varphi^{\sigma_j^g}}{\frac{\gamma_i^g}{\sigma_j^g - \gamma_i^g} \varphi^{\sigma_j^g - \gamma_i^g} \Big|_{\varphi_{ij}^{*gt}}^{\infty}} \right] dG_i^g(\varphi) \\ &= \int_{\varphi_{ij}^{*gt}}^{\infty} p_{ij}^{gt}(\varphi) \left[\frac{\varphi^{\sigma_j^g}}{\frac{\gamma_i^g}{\gamma_i^g - \sigma_j^g} (\varphi_{ij}^{*gt})^{\sigma_j^g - \gamma_i^g}} \right] dG_i^g(\varphi). \end{aligned} \tag{56}$$

Once again, using Equation (11) (twice) along with $G_i^g(\varphi) = 1 - \varphi^{-\gamma_i^g}$ and $\gamma_i^g > \sigma_j^g - 1$, we get:

$$\begin{aligned}
UV_{ij}^{gt} &= \frac{(\gamma_i^g - \sigma_j^g)}{\gamma_i^g (\varphi_{ij}^{*gt})^{\sigma_j^g - \gamma_i^g}} \int_{\varphi_{ij}^{*gt}}^{\infty} p_{ij}^{gt}(\varphi)\varphi^{\sigma_j^g} dG_i^g(\varphi) \\
&= \frac{(\gamma_i^g - \sigma_j^g)(1+t_{ij}^{gt})\tau_{ij}^{gt}\sigma_j^g z_i^{gt}}{\gamma_i^g(\sigma_j^g - 1)(\varphi_{ij}^{*gt})^{\sigma_j^g - \gamma_i^g}} \int_{\varphi_{ij}^{*gt}}^{\infty} \varphi^{\sigma_j^g - 1} dG_i^g(\varphi) \\
&= \frac{(\gamma_i^g - \sigma_j^g)(1+t_{ij}^{gt})\tau_{ij}^{gt}\sigma_j^g z_i^{gt}}{(\sigma_j^g - 1)(\varphi_{ij}^{*gt})^{\sigma_j^g - \gamma_i^g}} \int_{\varphi_{ij}^{*gt}}^{\infty} \varphi^{\sigma_j^g - \gamma_i^g - 2} d\varphi \\
&= \frac{(\gamma_i^g - \sigma_j^g)(1+t_{ij}^{gt})\tau_{ij}^{gt}\sigma_j^g z_i^{gt}}{(\sigma_j^g - \gamma_i^g - 1)(\sigma_j^g - 1)(\varphi_{ij}^{*gt})^{\sigma_j^g - \gamma_i^g}} \varphi^{\sigma_j^g - \gamma_i^g - 1}\Big|_{\varphi_{ij}^{*gt}}^{\infty} \\
&= \frac{(\gamma_i^g - \sigma_j^g)(1+t_{ij}^{gt})\tau_{ij}^{gt}\sigma_j^g z_i^{gt}}{(\gamma_i^g - \sigma_j^g + 1)(\sigma_j^g - 1)(\varphi_{ij}^{*gt})^{\sigma_j^g - \gamma_i^g}} (\varphi_{ij}^{*gt})^{\sigma_j^g - \gamma_i^g - 1}, \\
&= \frac{(\gamma_i^g - \sigma_j^g)(1+t_{ij}^{gt})\tau_{ij}^{gt}\sigma_j^g z_i^{gt}}{(\gamma_i^g - \sigma_j^g + 1)(\sigma_j^g - 1)\varphi_{ij}^{*gt}} \\
&= \frac{(\gamma_i^g - \sigma_j^g)}{(\gamma_i^g - \sigma_j^g + 1)} p_{ij}^{gt}(\varphi_{ij}^{*gt}).
\end{aligned} \tag{57}$$

A.2 Sato-Vartia price indexes for imported goods

Following Sato (1976), we can define the log-change of a price index ($P_{F_j}^{gt}$) for each good imported by country j from different trade partners i as

$$\ln\left(\frac{P_{F_j}^{gt}}{P_{F_j}^{gt-1}}\right) = \sum_{i=1,\, i\neq j}^{J} \ln\left[\left(\frac{\kappa_{ij}^{gt}}{\kappa_{ij}^{gt-1}}\right)^{\frac{1}{1-\sigma_j^g}} \frac{P_{ij}^{gt}}{P_{ij}^{gt-1}}\right]^{w_{ij}^{gt}}, \tag{58}$$

where w_{ij}^{gt} is the weight for origin i, such that $\sum_{i=1}^{J} w_{ij}^{gt} = 1$ for every $j \in J$ and $t = 2, ..., T$. Sato's (1976) ideal weight will therefore be given by:

$$w_{ij}^{gt} = \frac{(s_{gt}^{ij} - s_{gt-1}^{ij})/(\ln s_{gt}^{ij} - \ln s_{gt-1}^{ij})}{\sum_{k=1,\, k\neq j}^{J}(s_{gt}^{kj} - s_{gt-1}^{kj})/(\ln s_{gt}^{kj} - \ln s_{gt-1}^{kj})}$$

with

$$s_{gt}^{ij} = \frac{P_{ij}^{gt} C_{ij}^{gt}}{\sum_{k=1,\, k\neq j}^{J} P_{kj}^{gt} C_{kj}^{gt}} = \frac{V_{ij}^{g}}{V_{F_j}^{g}} = \kappa_{ij}^{gt}\left(\frac{P_{ij}^{gt}}{P_{F_j}^{gt}}\right)^{1-\sigma_j^g},$$

where the last equality follows directly from Equation (1).

Note that Equation (58) can also be written as:

$$\frac{P_{F_j}^{gt}}{P_{F_j}^{gt-1}} = \prod_{i=1,\, i\neq j}^{J} \left[\left(\frac{\kappa_{ij}^{gt}}{\kappa_{ij}^{gt-1}}\right)^{\frac{1}{1-\sigma_j^g}} \frac{P_{ij}^{gt}}{P_{ij}^{gt-1}}\right]^{w_{ij}^{gt}}. \tag{59}$$

Proceeding accordingly, we can write the corresponding relationship for the changes in unit prices of each good imported by country j as:

$$\frac{UV_{F_j}^{gt}}{UV_{F_j}^{gt-1}} = \prod_{i=1,\, i\neq j}^{J} \left(\frac{UV_{ij}^{gt}}{UV_{ij}^{gt-1}}\right)^{w_{ij}^{gt}}. \tag{60}$$

To achieve Equation (34), we use Equations (33) and (59) to rewrite this identity as:

$$\frac{UV_{F_j}^{gt}}{UV_{F_j}^{gt-1}} = \prod_{i=1,\, i\neq j}^{J} \left[\left(\frac{N_{ij}^{gt}}{N_{ij}^{gt-1}} \right)^{\frac{1}{\sigma_j^g - 1}} \frac{P_{ij}^{gt}}{P_{ij}^{gt-1}} \right]^{w_{ij}^{gt}}$$

$$= \left(\frac{P_{F_j}^{gt}}{P_{F_j}^{gt-1}} \right) \left[\prod_{i=1,\, i\neq j}^{J} \left(\frac{\kappa_{ij}^{gt}}{\kappa_{ij}^{gt-1}} \right)^{-\frac{1}{1-\sigma_j^g}} \left(\frac{N_{ij}^{gt}}{N_{ij}^{gt-1}} \right)^{\frac{1}{\sigma_j^g - 1}} \right]^{w_{ij}^{gt}} \tag{61}$$

$$= \left(\frac{P_{F_j}^{gt}}{P_{F_j}^{gt-1}} \right) \left(\frac{\kappa_{F_j}^{gt} N_{F_j}^{gt}}{\kappa_{F_j}^{gt-1} N_{F_j}^{gt-1}} \right)^{\frac{1}{\sigma_j^g - 1}}.$$

A.3 The micro demand curve

Multiplying both sides of Equation (35) by the Sato-Vartia weights w_{ij}^{gt} and summing across all origins $i \neq j$, we get:

$$\sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt} \left(\Delta \ln V_{ij}^{gt} - \Delta \ln V_{jj}^{gt} \right) = (1-\sigma_j^g) \left[\sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt} \Delta \ln UV_{ij}^{gt} \right] - (1-\sigma_j^g) \left[\sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt} \Delta \ln UV_{F_j}^{gt} \right] +$$
$$(1-\omega_j^g) \left[\sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt} \Delta \ln UV_{F_j}^{gt} \right] - (1-\omega_j^g) \left[\sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt} \Delta \ln UV_{jj}^{gt} \right] + \sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt} \varepsilon_{ij}^{gt}. \tag{62}$$

Subtracting Equation (35) from this result, we obtain:

$$\Delta \ln V_{ij}^{gt} - \Delta \ln V_{jj}^{gt} - \sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt} \left(\Delta \ln V_{ij}^{gt} - \Delta \ln V_{jj}^{gt} \right) = (1-\sigma_j^g) \left(\Delta \ln UV_{ij}^{gt} - \Delta \ln UV_{F_j}^{gt} \right) +$$
$$(1-\omega_j^g) \left(\Delta \ln UV_{F_j}^{gt} - \Delta \ln UV_{jj}^{gt} \right) + \varepsilon_{ij}^{gt} - (1-\sigma_j^g) \left[\sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt} \Delta \ln UV_{ij}^{gt} \right] +$$
$$(1-\sigma_j^g) \left[\sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt} \Delta \ln UV_{F_j}^{gt} \right] - (1-\omega_j^g) \left[\sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt} \Delta \ln UV_{F_j}^{gt} \right] + \tag{63}$$
$$(1-\omega_j^g) \left[\sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt} \Delta \ln UV_{jj}^{gt} \right] - \sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt} \varepsilon_{ij}^{gt}.$$

Since $V_{F_j}^{gt} = \sum_{i=1,\, i\neq j}^{J} V_{ij}^{gt}$ and $\sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt} V_{jj}^{gt} = V_{jj}^{gt} \sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt} = V_{jj}^{gt}$, the terms on the LHS can be simplified to:

$$\Delta \ln V_{ij}^{gt} - \Delta \ln V_{jj}^{gt} - \sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt} \left(\Delta \ln V_{ij}^{gt} - \Delta \ln V_{jj}^{gt} \right) = \Delta \ln \left(\frac{V_{ij}^{gt}}{V_{F_j}^{gt}} \right). \tag{64}$$

Proceeding accordingly, we can rewrite the terms on the RHS of Equation (63) as:

$$
\begin{aligned}
&(1-\sigma_j^g)\left(\Delta\ln UV_{ij}^{gt}\right)+\varepsilon_{ij}^{gt}-(1-\sigma_j^g)\left[\sum_{i=1,\,i\neq j}^{J} w_{ij}^{gt}\Delta\ln UV_{ij}^{gt}\right]+(1-\sigma_j^g)\left[\left(\sum_{i=1,\,i\neq j}^{J} w_{ij}^{gt}\right)-1\right]\Delta\ln UV_{F_j}^{gt}-\\
&(1-\omega_j^g)\left[\left(\sum_{i=1,\,i\neq j}^{J} w_{ij}^{gt}\right)-1\right]\Delta\ln UV_{F_j}^{gt}+(1-\omega_j^g)\left[\left(\sum_{i=1,\,i\neq j}^{J} w_{ij}^{gt}\right)-1\right]\Delta\ln UV_{jj}^{gt}-\sum_{i=1,\,i\neq j}^{J} w_{ij}^{gt}\varepsilon_{ij}^{gt}=\\
&(1-\sigma_j^g)\left(\Delta\ln UV_{ij}^{gt}\right)+\varepsilon_{ij}^{gt}-(1-\sigma_j^g)(\Delta\ln UV_{F_j}^{gt})-\sum_{i=1,\,i\neq j}^{J} w_{ij}^{gt}\varepsilon_{ij}^{gt}=\\
&(1-\sigma_j^g)\left(\Delta\ln UV_{ij}^{gt}-\Delta\ln UV_{F_j}^{gt}\right)+\Delta\ln\left(\frac{\kappa_{ij}^{gt}}{\kappa_{F_j}^{gt}}\right)+\Delta\ln\left(\frac{N_{ij}^{gt}}{N_{F_j}^{gt}}\right)+\Delta\ln\left(\frac{\beta_{F_j}^{g}}{\beta_{jj}^{g}}\right)-\frac{(1-\omega_j^g)}{(\sigma_j^g-1)}\Delta\ln\left(\frac{\kappa_{F_j}^{gt}N_{F_j}^{gt}}{N_{jj}^{gt}}\right)-\\
&\sum_{i=1,\,i\neq j}^{J} w_{ij}^{gt}\Delta\ln\kappa_{ij}^{gt}+\sum_{i=1,\,i\neq j}^{J} w_{ij}^{gt}\Delta\ln\kappa_{F_j}^{gt}-\sum_{i=1,\,i\neq j}^{J} w_{ij}^{gt}\Delta\ln N_{ij}^{gt}+\sum_{i=1,\,i\neq j}^{J} w_{ij}^{gt}\Delta\ln N_{F_j}^{gt}+\sum_{i=1,\,i\neq j}^{J}\Delta\ln\left(\frac{\beta_{F_j}^{g}}{\beta_{jj}^{g}}\right)-\\
&\frac{(1-\omega_j^g)}{(\sigma_j^g-1)}\sum_{i=1,\,i\neq j}^{J} w_{ij}^{gt}\Delta\ln\left(\kappa_{F_j}^{gt}N_{F_j}^{gt}\right)+\frac{(1-\omega_j^g)}{(\sigma_j^g-1)}\sum_{i=1,\,i\neq j}^{J} w_{ij}^{gt}\Delta\ln N_{jj}^{gt}=\\
&(1-\sigma_j^g)\left(\Delta\ln UV_{ij}^{gt}-\Delta\ln UV_{F_j}^{gt}\right)-\Delta\ln\kappa_{ij}^{gt}+\left[\left(\sum_{i=1,\,i\neq j}^{J} w_{ij}^{gt}\right)-1\right]\Delta\ln\kappa_{F_j}^{gt}-\Delta\ln N_{ij}^{gt}+\\
&\left[\left(\sum_{i=1,\,i\neq j}^{J} w_{ij}^{gt}\right)-1\right]\Delta\ln N_{F_j}^{gt}-\left[\left(\sum_{i=1,\,i\neq j}^{J} w_{ij}^{gt}\right)-1\right]\Delta\ln\left(\frac{\beta_{F_j}^{g}}{\beta_{jj}^{g}}\right)+\\
&\frac{(1-\omega_j^g)}{(\sigma_j^g-1)}\left[\left(\sum_{i=1,\,i\neq j}^{J} w_{ij}^{gt}\right)-1\right]\Delta\ln\left(\frac{\kappa_{F_j}^{gt}N_{F_j}^{gt}}{N_{jj}^{gt}}\right)-\Delta\ln\kappa_{F_j}^{gt}-\Delta\ln N_{F_j}^{gt}=\\
&(1-\sigma_j^g)\left(\Delta\ln UV_{ij}^{gt}-\Delta\ln UV_{F_j}^{gt}\right)+\Delta\ln\left(\frac{\kappa_{ij}^{gt}}{\kappa_{F_j}^{gt}}\right)+\Delta\ln\left(\frac{N_{ij}^{gt}}{N_{F_j}^{gt}}\right).
\end{aligned}
\tag{65}
$$

Plugging Equations (64) and (65) back to the LHS and RHS of Equation (63), we get the micro demand curve defined in Equation (37).

A.4 The macro demand curve

Combining Equations (2) and (3) and summing across all origins $i \neq j$, we get:

$$
V_{F_j}^{gt}=\sum_{i=1,\,i\neq j}^{J} V_{ij}^{gt}=\sum_{i=1,\,i\neq j}^{J}\left[\kappa_{ij}^{gt}\left(\frac{P_{ij}^{gt}}{P_{F_j}^{gt}}\right)^{1-\sigma_j^g}\beta_{F_j}^{gt}\left(\frac{P_{F_j}^{gt}}{P_j^{gt}}\right)^{1-\omega_j^g}V_j^{gt}\right]. \tag{66}
$$

Noting that $\sum_{i=1,\,i\neq j}^{J}\kappa_{ij}^{gt}(P_{ij}^{gt})^{1-\sigma_j^g}=\left(P_{F_j}^{gt}\right)^{(1-\sigma_j^g)}$ (see Equation (58)) and imposing $\sum_{i=1,\,i\neq j}^{J}\kappa_{ij}^{gt}=1$, we can reduce Equation (66) to:

$$
V_{F_j}^{gt}=\beta_{F_j}^{gt}\left(\frac{P_{F_j}^{gt}}{P_j^{gt}}\right)^{1-\omega_j^g}V_j^{gt}.
$$

Using the definition of domestic expenditure (Equation (1)), we can rewrite this expression as:

$$\frac{V_{F_j}^{gt}}{V_{jj}^{gt}} = \left(\frac{\beta_{F_j}^{gt}}{\beta_{jj}^{gt}}\right)\left(\frac{P_{F_j}^{gt}}{P_{jj}^{gt}}\right)^{1-\omega_j^g}.$$

Taking log-differences of each term and assuming $\omega_j^g = \omega_j$ for a given set of products (i.e., for $g \in \{1...G^s\}$), we get:

$$\Delta \ln\left(\frac{V_{F_j}^{gt}}{V_{jj}^{gt}}\right) = -(\omega_j - 1)\Delta \ln\left(\frac{P_{F_j}^{gt}}{P_{jj}^{gt}}\right) + \Delta \ln\left(\frac{\beta_{F_j}^{gt}}{\beta_{jj}^{gt}}\right).$$

In terms of the unit values defined in Equations (33) and (34), this expression becomes:

$$\Delta \ln\left(\frac{V_{F_j}^{gt}}{V_{jj}^{gt}}\right) = -(\omega_j - 1)\Delta \ln\left(\frac{UV_{F_j}^{gt}}{UV_{jj}^{gt}}\right) + \Delta \ln\left(\frac{\beta_{F_j}^{gt}}{\beta_{jj}^{gt}}\right) + \left(\frac{\omega_j - 1}{\sigma_j^g - 1}\right)\left[\Delta \ln \kappa_{F_j}^{gt} + \Delta \ln\left(\frac{N_{F_j}^{gt}}{N_{jj}^{gt}}\right)\right].$$

Shifting the unit values to the left and reorganizing the terms, we obtain Equation (38).

A.5 Employing IV with source-country indicators to pool u_{iF}^{gt} over trade-partners

Combining Equations (44) and (45), we get:

$$\hat{\boldsymbol{\theta}}^{\mathbf{g}} = \left[\left(\hat{\mathbf{X}}_{\mathbf{1S}}^{\mathbf{g}}\right)^{\mathbf{T}} \hat{\mathbf{X}}_{\mathbf{1S}}^{\mathbf{g}}\right]^{-1}\left(\hat{\mathbf{X}}_{\mathbf{1S}}^{\mathbf{g}}\right)^{\mathbf{T}} \mathbf{Y}^{\mathbf{g}} =$$
$$\left\{(\mathbf{X}^{\mathbf{g}})^{\mathbf{T}} \mathbf{Z}^{\mathbf{g}}\left[(\mathbf{Z}^{\mathbf{g}})^{\mathbf{T}} \mathbf{Z}^{\mathbf{g}}\right]^{-1} (\mathbf{Z}^{\mathbf{g}})^{\mathbf{T}} \mathbf{X}^{\mathbf{g}}\right\}^{-1} (\mathbf{X}^{\mathbf{g}})^{\mathbf{T}} \mathbf{Z}^{\mathbf{g}}\left[(\mathbf{Z}^{\mathbf{g}})^{\mathbf{T}} \mathbf{Z}^{\mathbf{g}}\right]^{-1} (\mathbf{Z}^{\mathbf{g}})^{\mathbf{T}} \mathbf{Y}^{\mathbf{g}}.$$

Using Equation (43), this expression becomes:

$$\begin{aligned}\hat{\boldsymbol{\theta}}^{\mathbf{g}} = &\left\{(\mathbf{X}^{\mathbf{g}})^{\mathbf{T}} \mathbf{Z}^{\mathbf{g}}\left[(\mathbf{Z}^{\mathbf{g}})^{\mathbf{T}} \mathbf{Z}^{\mathbf{g}}\right]^{-1} (\mathbf{Z}^{\mathbf{g}})^{\mathbf{T}} \mathbf{X}^{\mathbf{g}}\right\}^{-1} (\mathbf{X}^{\mathbf{g}})^{\mathbf{T}} \mathbf{Z}^{\mathbf{g}}\left[(\mathbf{Z}^{\mathbf{g}})^{\mathbf{T}} \mathbf{Z}^{\mathbf{g}}\right]^{-1} (\mathbf{Z}^{\mathbf{g}})^{\mathbf{T}} \mathbf{X}^{\mathbf{g}}\boldsymbol{\theta}^{\mathbf{g}} + \\ &\left\{(\mathbf{X}^{\mathbf{g}})^{\mathbf{T}} \mathbf{Z}^{\mathbf{g}}\left[(\mathbf{Z}^{\mathbf{g}})^{\mathbf{T}} \mathbf{Z}^{\mathbf{g}}\right]^{-1} (\mathbf{Z}^{\mathbf{g}})^{\mathbf{T}} \mathbf{X}^{\mathbf{g}}\right\}^{-1} (\mathbf{X}^{\mathbf{g}})^{\mathbf{T}} \mathbf{Z}^{\mathbf{g}}\left[(\mathbf{Z}^{\mathbf{g}})^{\mathbf{T}} \mathbf{Z}^{\mathbf{g}}\right]^{-1} (\mathbf{Z}^{\mathbf{g}})^{\mathbf{T}} \mathbf{u}^{\mathbf{g}}.\end{aligned} \tag{67}$$

Note that our first-stage instruments $\mathbf{Z}^{\mathbf{g}}$ are defined as:

$$\mathbf{Z}^{\mathbf{g}} = \begin{bmatrix} \mathbf{e}_{\mathbf{T}_1^{\mathbf{g}}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{e}_{\mathbf{T}_i^{\mathbf{g}}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{e}_{\mathbf{T}_J^{\mathbf{g}}} \end{bmatrix}, \tag{68}$$

where $\mathbf{e}_{\mathbf{T}_i^{\mathbf{g}}}$ are column vectors of 1's of length T_i^g designating our indicator variables for each trade partner $i \neq j$.

Thus, we can rewrite Equation (67) as:

$$\hat{\boldsymbol{\theta}}^{\mathbf{g}} = \boldsymbol{\theta}^{\mathbf{g}} +$$

$$\left\{ (\mathbf{X^g})^{\mathbf{T}} \mathbf{Z^g} \left[(\mathbf{Z^g})^{\mathbf{T}} \mathbf{Z^g} \right]^{-\mathbf{1}} (\mathbf{Z^g})^{\mathbf{T}} \mathbf{X^g} \right\}^{-\mathbf{1}} (\mathbf{X^g})^{\mathbf{T}} \mathbf{Z^g} \left[(\mathbf{Z^g})^{\mathbf{T}} \mathbf{Z^g} \right]^{-\mathbf{1}} \begin{bmatrix} \left(\mathbf{e_{T_1^g}}\right)^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \left(\mathbf{e_{T_i^g}}\right)^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \left(\mathbf{e_{T_J^g}}\right)^T \end{bmatrix} \begin{bmatrix} \mathbf{u_1^g} \\ \vdots \\ \mathbf{u_i^g} \\ \vdots \\ \mathbf{u_J^g} \end{bmatrix} =$$

$$\boldsymbol{\theta}^{\mathbf{g}} + \left\{ (\mathbf{X^g})^{\mathbf{T}} \mathbf{Z^g} \left[(\mathbf{Z^g})^{\mathbf{T}} \mathbf{Z^g} \right]^{-\mathbf{1}} (\mathbf{Z^g})^{\mathbf{T}} \mathbf{X^g} \right\}^{-\mathbf{1}} (\mathbf{X^g})^{\mathbf{T}} \mathbf{Z^g} \left[(\mathbf{Z^g})^{\mathbf{T}} \mathbf{Z^g} \right]^{-\mathbf{1}} \begin{bmatrix} \sum_{t=1}^{T_1^g} u_1^{gt} \\ \vdots \\ \sum_{t=1}^{T_i^g} u_i^{gt} \\ \vdots \\ \sum_{t=1}^{T_J^g} u_J^{gt} \end{bmatrix}.$$

Assuming that the sum of the residuals is zero across t (in expectation) for each trade partner (Equation (40)), we thus have:

$$\operatorname*{plim}_{T \to \infty,\, i \neq j} \hat{\boldsymbol{\theta}}^{\mathbf{g}} = \boldsymbol{\theta}^{\mathbf{g}}.$$

A.6 Identification through heteroskedasticity

We should first note the columns of matrix $\hat{\mathbf{X}}^{\mathbf{g}}_{\mathbf{1S}}$ (resulting from the first step of the 2SLS procedure) are constituted of grouped means of the original $\mathbf{X^g}$'s.

To see this, notice that using our definition for $\mathbf{Z^g}$ in Equation (68) along with Equation (68), we can rewrite our predicted regressors from the first step as:

$$\hat{\mathbf{X}}^{\mathbf{g}}_{\mathbf{1S}} = \mathbf{Z}^{\mathbf{g}} \left(\begin{bmatrix} \left(\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{1}}}\right)^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \left(\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{i}}}\right)^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \left(\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{J}}}\right)^T \end{bmatrix} \begin{bmatrix} \mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{1}}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{i}}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{J}}} \end{bmatrix} \right)^{-\mathbf{1}} \left(\mathbf{Z}^{\mathbf{g}}\right)^{\mathbf{T}} \mathbf{X}^{\mathbf{g}} =$$

$$\mathbf{Z}^{\mathbf{g}} \begin{bmatrix} 1/T^g_1 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 1/T^g_i & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 1/T^g_J \end{bmatrix} \left(\mathbf{Z}^{\mathbf{g}}\right)^{\mathbf{T}} \mathbf{X}^{\mathbf{g}} = \begin{bmatrix} \mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{1}}}/T^g_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{i}}}/T^g_i & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{J}}}/T^g_J \end{bmatrix} \left(\mathbf{Z}^{\mathbf{g}}\right)^{\mathbf{T}} \mathbf{X}^{\mathbf{g}} =$$

$$\begin{bmatrix} \mathbf{J}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{1}}}/T^g_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{J}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{i}}}/T^g_i & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{J}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{J}}}/T^g_J \end{bmatrix} \mathbf{X}^{\mathbf{g}} = \begin{bmatrix} \mathbf{J}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{1}}}/T^g_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{J}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{i}}}/T^g_i & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{J}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{J}}}/T^g_J \end{bmatrix} \begin{bmatrix} \mathbf{X}^{\mathbf{g}}_{\mathbf{11F}} & \mathbf{X}^{\mathbf{g}}_{\mathbf{21F}} \\ \vdots & \vdots \\ \mathbf{X}^{\mathbf{g}}_{\mathbf{1iF}} & \mathbf{X}^{\mathbf{g}}_{\mathbf{2iF}} \\ \vdots & \vdots \\ \mathbf{X}^{\mathbf{g}}_{\mathbf{1JF}} & \mathbf{X}^{\mathbf{g}}_{\mathbf{2JF}} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{\sum_t X^{gt}_{11F}}{T^g_1}\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{1}}} & \frac{\sum_t X^{gt}_{21F}}{T^g_1}\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{1}}} \\ \vdots & \vdots \\ \frac{\sum_t X^{gt}_{1iF}}{T^g_i}\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{i}}} & \frac{\sum_t X^{gt}_{2iF}}{T^g_i}\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{i}}} \\ \vdots & \vdots \\ \frac{\sum_t X^{gt}_{1JF}}{T^g_J}\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{J}}} & \frac{\sum_t X^{gt}_{2JF}}{T^g_J}\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{J}}} \end{bmatrix} = \begin{bmatrix} \bar{X}^g_{11F}\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{1}}} & \bar{X}^g_{21F}\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{1}}} \\ \vdots & \vdots \\ \bar{X}^g_{1iF}\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{i}}} & \bar{X}^g_{2iF}\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{i}}} \\ \vdots & \vdots \\ \bar{X}^g_{1JF}\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{J}}} & \bar{X}^g_{2JF}\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{J}}} \end{bmatrix}. \tag{69}$$

where $\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{i}}}$ are column vectors of 1's of length T^g_i and $\mathbf{J}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{i}}}$ are square matrices of size $T^g_i \times T^g_i$.

We should further note that, using Equations (37), (41) and the definitions in Equation (43), we are able to write X^{gt}_{1iF} and X^{gt}_{2iF} in terms of ε^{gt}_{iF} and δ^{gt}_{iF} as:

$$\begin{aligned} X^{gt}_{1iF} &= \left[\Delta \ln\left(\frac{V^{gt}_{ij}}{V^{gt}_{F_j}}\right)\right]^2 = \left[(1-\rho^g_{1j})\varepsilon^{gt}_{iF} + (\sigma^g_j - 1)\delta^{gt}_{iF}\right]^2 = \\ &(1-\rho^g_{1j})^2 \left(\varepsilon^{gt}_{iF}\right)^2 - 2(1-\rho^g_{1j})(\sigma^g_j - 1)\varepsilon^{gt}_{iF}\delta^{gt}_{iF} + (\sigma^g_j - 1)^2 \left(\delta^{gt}_{iF}\right)^2. \end{aligned} \tag{70}$$

and

$$\begin{aligned} X^{gt}_{2iF} &= \left[\Delta \ln\left(\frac{UV^{gt}_{ij}}{UV^{gt}_{F_j}}\right)\right]\left[\Delta \ln\left(\frac{V^{gt}_{ij}}{V^{gt}_{F_j}}\right)\right] = \\ &\frac{\rho^g_{1j}(1-\rho^g_{1j})}{(\sigma^g_j - 1)} \left(\varepsilon^{gt}_{iF}\right)^2 + (1-2\rho^g_{1j})\varepsilon^{gt}_{iF}\delta^{gt}_{iF} - (\sigma^g_j - 1)\left(\delta^{gt}_{iF}\right)^2. \end{aligned} \tag{71}$$

Using Equation (69) and assuming $E\left[\varepsilon^{gt}_{iF}\right] = 0$, $E\left[\delta^{gt}_{iF}\right] = 0$, $E\left[\sum_t \varepsilon^{gt}_{iF}\delta^{gt}_{iF}\right] = 0$ for each $i \neq j$ (as in Equation (40)), and ε^{gt}_{iF} and δ^{gt}_{iF} stationary with variances σ_{ε_i} and σ_{δ_i}, we can thus conclude that $\hat{\mathbf{X}}^{\mathbf{g}}_{\mathbf{1S}}$ will asymptotically converge to:

$$\underset{T\to\infty,\, i\neq j}{\text{plim}} \hat{\mathbf{X}}^{\mathbf{g}}_{\mathbf{1S}} = \begin{bmatrix} \left[(1-\rho^g_{1j})^2\sigma^2_{\varepsilon_1} + (\sigma^g_j - 1)^2\sigma^2_{\delta_1}\right]\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{1}}} & \left[\frac{\rho^g_{1j}(1-\rho^g_{1j})}{(\sigma^g_j-1)}\sigma^2_{\varepsilon_1} - (\sigma^g_j - 1)\sigma^2_{\delta_1}\right]\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{1}}} \\ \vdots & \vdots \\ \left[(1-\rho^g_{1j})^2\sigma^2_{\varepsilon_i} + (\sigma^g_j - 1)^2\sigma^2_{\delta_i}\right]\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{i}}} & \left[\frac{\rho^g_{1j}(1-\rho^g_{1j})}{(\sigma^g_j-1)}\sigma^2_{\varepsilon_i} - (\sigma^g_j - 1)\sigma^2_{\delta_i}\right]\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{i}}} \\ \vdots & \vdots \\ \left[(1-\rho^g_{1j})^2\sigma^2_{\varepsilon_J} + (\sigma^g_j - 1)^2\sigma^2_{\delta_J}\right]\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{J}}} & \left[\frac{\rho^g_{1j}(1-\rho^g_{1j})}{(\sigma^g_j-1)}\sigma^2_{\varepsilon_J} - (\sigma^g_j - 1)\sigma^2_{\delta_J}\right]\mathbf{e}_{\mathbf{T}^{\mathbf{g}}_{\mathbf{J}}} \end{bmatrix}. \tag{72}$$

To avoid multicollinearity and be able to retrieve separate coefficients for $\boldsymbol{\theta}^{\mathbf{g}}_{\mathbf{1}}$ and $\boldsymbol{\theta}^{\mathbf{g}}_{\mathbf{2}}$ in Equation (43), we must guarantee that $\hat{\mathbf{X}}^{\mathbf{g}}_{\mathbf{1S}}$ is full rank. In other words, since our intention is to estimate $\hat{\boldsymbol{\theta}}^{\mathbf{g}}_{\mathbf{1}}$ and $\hat{\boldsymbol{\theta}}^{\mathbf{g}}_{\mathbf{2}}$, we must have at least one $[2\times 2]$ submatrix of $\hat{\mathbf{X}}^{\mathbf{g}}_{\mathbf{1S}}$ with a non-zero determinant. Note that taking two separate rows i and j $(i \neq j)$ from $\boldsymbol{\theta}^{\mathbf{g}}_{\mathbf{1}}$, this is equivalent to have:

$$\begin{gathered} (1-\rho^g_{1j})^2(\sigma^g_j - 1)\sigma^2_{\varepsilon_i}\sigma^2_{\delta_j} + \rho^g_{1j}(1-\rho^g_{1j})(\sigma^g_j-1)\sigma^2_{\varepsilon_j}\sigma^2_{\delta_i} \neq \\ (1-\rho^g_{1j})^2(\sigma^g_j - 1)\sigma^2_{\varepsilon_j}\sigma^2_{\delta_i} + \rho^g_{1j}(1-\rho^g_{1j})(\sigma^g_j-1)\sigma^2_{\varepsilon_i}\sigma^2_{\delta_j} \\ \Leftrightarrow \\ \frac{\sigma^2_{\varepsilon_i}}{\sigma^2_{\varepsilon_j}} \neq \frac{\sigma^2_{\delta_i}}{\sigma^2_{\delta_j}}. \end{gathered} \tag{73}$$

A.7 Adding information with extra moment conditions

We will first show that the moment conditions of Equation (51) add information to those of Equations (40) and (42).

To check the type of restrictions that are being provided by Equations (40) and (42), we combine the supply curves that back these assumptions (i.e., Equations (39) and (41), to get[40]:

$$\begin{aligned} \Delta\ln\left(\frac{UV^{gt}_{ij}}{UV^{gt}_{F_j}}\right) &= \rho^g_{1j}\left(\frac{\varepsilon^{gt}_{iF}}{\sigma^g_j - 1}\right) + \delta^{gt}_{iF} - \rho^g_{2j}\frac{(\omega_j - 1)}{(\sigma^g_j - 1)}\Delta\ln\left(\frac{UV^{gt}_{F_j}}{UV^{gt}_{jj}}\right) + \rho^g_{2j}\rho_{F_j}\left(\frac{\varepsilon^{gt}_{F_j}}{\sigma^g_j - 1}\right) + \rho^g_{2j}\frac{(\omega_j - 1)}{(\sigma^g_j - 1)}\delta^{gt}_{F_j} \\ &= \rho^g_{1j}\left(\frac{\varepsilon^{gt}_{ij}}{\sigma^g_j - 1}\right) - \rho^g_{2j}\frac{(\omega_j - 1)}{(\sigma^g_j - 1)}\Delta\ln\left(\frac{UV^{gt}_{F_j}}{UV^{gt}_{jj}}\right) + \left(\rho^g_{2j}\rho_{F_j} - \rho^g_{1j}\right)\left(\frac{\varepsilon^{gt}_{F_j}}{\sigma^g_j - 1}\right) + \delta^{gt}_{iF} + \rho^g_{2j}\frac{(\omega_j - 1)}{(\sigma^g_j - 1)}\delta^{gt}_{F_j}, \end{aligned} \tag{74}$$

where the last equality follows from the identity $\varepsilon^{gt}_{ij} = \varepsilon^{gt}_{iF} + \varepsilon^{gt}_{F_j}$ (see Equations (35), (37) and (38)).

In addition, again using Equations (39) and (41), along with Equation (50), we can rewrite the supply shock δ^{gt}_{ij} as:

$$\begin{aligned} \delta^{gt}_{ij} &= \delta^{gt}_{iF} - \hat{\rho}^g_{1j}\frac{\varepsilon^{gt}_{F_j}}{(\hat{\sigma}^g_j - 1)} + \rho^g_{2j}\frac{(\omega_j - 1)}{(\hat{\sigma}^g_j - 1)}\Delta\ln\left(\frac{UV^{gt}_{F_j}}{UV^{gt}_{jj}}\right) \\ &= \delta^{gt}_{iF} + \rho^g_{2j}\frac{(\omega_j - 1)}{(\hat{\sigma}^g_j - 1)}\delta^{gt}_{F_j} + \left(\rho^g_{2j}\rho_{F_j} - \hat{\rho}^g_{1j}\right)\left(\frac{\varepsilon^{gt}_{F_j}}{\sigma^g_j - 1}\right) \end{aligned} \tag{75}$$

Notice that if we impose $\rho^g_{2j}\rho_{F_j} = \hat{\rho}^g_{1j}$, the above expression is reduced to $\delta^{gt}_{ij} = \delta^{gt}_{iF} + \rho^g_{2j}\frac{(\omega_j-1)}{(\hat{\sigma}^g_j-1)}\delta^{gt}_{F_j}$ and Equation (74) becomes Equation (50). Thus, Equation (51) is imposing an extra restriction ($\rho^g_{2j}\rho_{F_j} = \hat{\rho}^g_{1j}$) to the moments that were originally retrieved from Equations (40) and (42).

[40]Note that we multiply Equation (41) by $\rho^g_{2j}(\omega_j - 1)/(\sigma^g_j - 1)$ before coupling it to Equation (39).

Conversely, we can also show that Equation (42) adds information to Equation (51). Note that if we multiply the demand shocks ε_{ij}^{gt} in Equation (37) by the Sato-Vartia weights w_{ij}^{gt} and sum over trade partners ($i \neq j$) we get:

$$\sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt}\varepsilon_{ij}^{gt} = \sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt}\left[\Delta\ln\left(\frac{\kappa_{ij}^{gt}}{\kappa_{F_j}^{gt}}\right) + \Delta\ln\left(\frac{N_{ij}^{gt}}{N_{F_j}^{gt}}\right) + \Delta\ln\left(\frac{\beta_{F_j}^{g}}{\beta_{jj}^{g}}\right) - \frac{(1-\omega_j^g)}{(\sigma_j^g-1)}\Delta\ln\left(\frac{\kappa_{F_j}^{gt}N_{F_j}^{gt}}{N_{jj}^{gt}}\right)\right] =$$
$$\Delta\ln\left(\frac{\sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt}\kappa_{ij}^{gt}}{\kappa_{F_j}^{gt}}\right) + \Delta\ln\left(\frac{\sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt}N_{ij}^{gt}}{N_{F_j}^{gt}}\right) + \Delta\ln\left(\frac{\beta_{F_j}^{g}}{\beta_{jj}^{g}}\right) - \frac{(1-\omega_j^g)}{(\sigma_j^g-1)}\Delta\ln\left(\frac{\kappa_{F_j}^{gt}N_{F_j}^{gt}}{N_{jj}^{gt}}\right) =$$
$$\Delta\ln\left(\frac{\beta_{F_j}^{g}}{\beta_{jj}^{g}}\right) - \frac{(1-\omega_j^g)}{(\sigma_j^g-1)}\Delta\ln\left(\frac{\kappa_{F_j}^{gt}N_{F_j}^{gt}}{N_{jj}^{gt}}\right) = \varepsilon_{F_j}^{gt}, \tag{76}$$

where the last equality follows from Equation (38).

Similarly, if we proceed accordingly with the supply shocks in Equation (50), we get:

$$\sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt}\delta_{ij}^{gt} = \sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt}\left[\Delta\ln\left(\frac{UV_{ij}^{gt}}{UV_{F_j}^{gt}}\right) - \hat{\rho}_{1j}^{g}\frac{\varepsilon_{ij}^{gt}}{(\hat{\sigma}_j^g-1)} + \rho_{2j}^{g}\frac{(\omega_j-1)}{(\hat{\sigma}_j^g-1)}\Delta\ln\left(\frac{UV_{F_j}^{gt}}{UV_{jj}^{gt}}\right)\right] =$$
$$\Delta\ln\left(\frac{\sum_{i=1,\, i\neq j}^{J} w_{ij}^{gt}UV_{ij}^{gt}}{UV_{F_j}^{gt}}\right) - \hat{\rho}_{1j}^{g}\frac{\varepsilon_{F_j}^{gt}}{(\hat{\sigma}_j^g-1)} + \rho_{2j}^{g}\rho_{F_j}\left(\frac{\varepsilon_{F_j}^{gt}}{\hat{\sigma}_j^g-1}\right) + \rho_{2j}^{g}\frac{(\omega_j-1)}{(\hat{\sigma}_j^g-1)}\delta_{F_j}^{gt} = \rho_{2j}^{g}\frac{(\omega_j-1)}{(\hat{\sigma}_j^g-1)}\delta_{F_j}^{gt} \tag{77}$$

where we again use $\rho_{2j}^{g}\rho_{F_j} = \hat{\rho}_{1j}^{g}$, along with Equations (41) and (76), to achieve the last identities.

Using Equations (76) and (76), we can now rewrite the moment conditions of Equation (42) as:

$$0 = E\left[\sum_t \varepsilon_{F_j}^{gt}\delta_{F_j}^{gt}\right] = E\left[\frac{(\hat{\sigma}_j^g-1)}{\rho_{2j}^{g}(\omega_j-1)}\sum_t\sum_{i\neq j}(w_{ij}^{gt})^2\varepsilon_{ij}^{gt}\delta_{ij}^{gt}\right] + E\left[\frac{(\hat{\sigma}_j^g-1)}{\rho_{2j}^{g}(\omega_j-1)}\sum_t\sum_{i\neq j,\, k} w_{ij}^{gt}w_{kj}^{gt}\varepsilon_{ij}^{gt}\delta_{ij}^{gt}\varepsilon_{kj}^{gt}\delta_{kj}^{gt}\right]$$

for all $g \in \{1\ldots G^s\}$.

Notice that the unweighted moment conditions of Equation (51) might eventually bring the first expectation of this expression to zero (or close to zero). However, the second expectation above imposes a new set of restrictions to those of Equation (51), meaning that the moments of (42) are indeed adding information to those of Equation (51).

B Data processing details

Columns (1) to (4) of Table 7 illustrate the sequential process of data cleaning and exclusions due to issues encountered in the matching and analysis of production and trade databases. Each column shows the number of remaining products by sector after addressing the following issues: (1) Multiple product codification mappings from trade to production databases; (2) Exclusion of products from PIA-Produto database with less than three firms' survey responses to ensure representative pricing; (3) Exclusion of products with fewer than 50 database observations (following FLOR); (4) Exclusion of goods yielding negative values for σ in more than 75% of bootstrap replications (as in FLOR). The final column presents the proportion of final remaining products' import value over the total import value for each sector.

Table 7: Summary of data processing and exclusions by sector.

Sector	Total PIA/PAM Prod.	Number of remaining products by sector (1) Codif. Match.	(2) Firm Resp. (≥ 3)	(3) Obs. Count (≥ 50)	(4) BS Filt. (>75% $\sigma < 0$)	Final Import Value (%)
Crop and Animal Production	47	43	41	15	15	50.02
Oil and Gas	5	4	1	1	1	59.01
Mining Extraction (Metal Ores)	33	15	11	6	6	69.45
Mining Extraction (Other)	70	36	26	19	18	12.60
Food Products	361	144	134	81	79	57.12
Beverages and Tobacco	45	17	15	13	13	83.63
Textiles	154	78	72	66	62	61.24
Wearing Apparel	94	35	31	29	28	19.08
Leather and Footwear	88	17	10	8	8	3.95
Wood Products	50	25	16	12	11	9.44
Paper and Paper Products	97	36	28	26	26	30.73
Printing and Reproduction of Recorded Media	60	8	1	1	1	0.15
Coke and Refined Petroleum Products	60	26	16	10	9	26.00
Chemical Products	530	433	348	282	280	48.24
Pharmaceuticals and Medicinal Chemical Products	135	35	18	11	10	4.77
Rubber and Plastic Products	123	46	42	38	38	45.79
Non-metallic Mineral Products	131	83	70	60	59	76.91
Basic Metals	141	86	67	58	55	43.91
Fabricated metal products	214	115	88	78	71	68.40
Computer and Electronic Products	178	119	88	83	71	57.55
Electrical Equipment	155	103	91	86	78	69.22
Machinery and Equipment (n.e.c)	424	262	218	193	138	32.21
Motor Vehicles, Trailers, and Semi-trailers	98	35	27	23	17	82.43
Other Transport Equipment	77	46	21	6	6	2.07
Other Manufacturing	233	95	44	41	40	47.24
All Sectors	3603	1942	1524	1246	1140	44.69

C Additional information about the simulations

C.1 Solution method

Balistreri et al. (2011) and Balistreri and Rutherford (2013) present a decomposition algorithm that addresses the computational challenge of solving a non-linear equilibrium model with multiple regions, factors, and commodities. The algorithm consists of two modules: a partial equilibrium (PE) model and a constant-returns general equilibrium (GE) model. The PE model captures the industrial organization in some sectors and the associated impact on productivity and prices, while the GE model determines relative prices and income. The models are iterated in policy simulations, with the PE module determining the industrial structure and the GE model establishing regional incomes and relative costs. The industrial structure is passed from the PE to the GE module, and the structure of aggregate demand is passed back from the GE to the PE module, until the models are mutually consistent and all conditions are satisfied. The authors argue that by using the divide-and-conquer procedure, numerical issues that arise in quantitative analyses using models such as Melitz (2003) are avoided.

The PE model is derived by removing and replacing certain equations in the GE model. Specifically, the equations for the unit cost of the input basket (Equation 8) and the total expenditure on good g in region j (Equation 24) are substituted with a reduced-form equation of constant elasticities for supply and demand, as follows

$$Y_i^g = \overline{Y}_i^g \left(\frac{z_i^g}{\overline{z}_i^g} \right)^{\mu} \tag{78}$$

and

$$V_j^g = \overline{V}_j^g \left(\frac{P_j^g}{\overline{P}_j^g} \right)^{1-\eta}, \tag{79}$$

where $\mu > 0$ and $\eta > 0$ are the price elasticities of supply and demand, respectively.[41] The variables $\overline{Y}_i^g$, $\overline{z}_i^g$, $\overline{V}_j^g$, and $\overline{P}_j^g$ represent the initial or benchmark values of the variables Y_i^g, z_i^g, V_j^g, and P_j^g, respectively. Furthermore, in the PE model, no equations related to income and factor markets are taken into account.

In the GE model with constant returns to scale, the equations for zero cutoff productivity, free entry condition, bilateral average firm productivity, and demand for variety φ are disregarded. Additionally, the bilateral price equations and market equilibrium equations for goods are replaced with

$$P_{ij}^g = \tau_{ij}^g(1 + t_{ij}^g)z_{ij}^g \tag{80}$$

and

$$Y_i^g = \sum_{j=1}^{N} \frac{\tau_{ij}^g V_{ij}^g}{P_{ij}^g}. \tag{81}$$

With these modifications, the GE model has a traditional Armington structure (Balistreri and Rutherford, 2013).

Given the definitions of the PE and GE models, Table 8 presents the variables and equations that comprise the (non-linear) system for each module. All variables that depend on productivity are considered in terms of the average firm.

Before detailing the four steps of the solution algorithm, it is worth noting that the Melitz structure was assumed for all sectors of the extractive and manufacturing industries. That is, a conventional Armington structure was assumed for the agriculture and services sectors. Regarding the services sector, we have chosen not to assume a Melitz structure, as this sector has a high share of input consumption from its own sector. This can exacerbate the issue of infinitely large effects, which would hinder the applicability of the model. For instance, Jafari and Britz (2018) assume perfect competition for sectors that use at least 20% of their own output as inputs.

The solution algorithm can be detailed in four steps. In the first step, the PE model is solved for each sector with assumed heterogeneous firm structure. The solution of this model results in new values for the variables Y_i^g, z_i^g, V_j^g, and P_j^g, as well as trade variables (V_{jj}^g, $V_{F_j}^g$, V_{ij}^g). The second step consists of recalibrating the parameters β_{jj}^g, $\beta_{F_j}^g$ and κ_{ij}^g using equations 1, 2, 3 and 80 with the values obtained in the first step.[42] In the third step, the GE model is solved, and new values for the variables Y_i^g, z_i^g, V_j^g, and P_j^g are obtained. In the fourth step, the solution values from the GE model are used to update the benchmark variables $\overline{Y}_i^g$, $\overline{z}_i^g$, $\overline{V}_j^g$, and $\overline{P}_j^g$. In this fourth step, the benchmark variable values of the PE model are compared with the values obtained from the GE model. If the difference is significant, the benchmark values are updated, and the algorithm returns to step 1. Otherwise, convergence is considered achieved.

C.2 Armington elasticities and shape parameter for firm size distribution

The values of the parameters σ^g, ω^g, and ζ^g used in the simulations are presented in Table 9. In addition to our estimates, we also list other estimates that can be found in the literature. We use 2GMM estimates for σ^g and ω^g if the lower limit of the confidence interval is greater than one. Otherwise, we use the 2SLS estimate by checking the same rule. If we don't have a valid estimate for a sector, we use the "All Sectors" valid estimates. In cases where the original estimates are at a more disaggregated level than that used in the simulations, we choose to use the simple average of the estimates belonging to the same sector of Table 9.

C.3 Additional results

[41] The demand function is expressed in terms of expenditure rather than quantities.

[42] See Balistreri and Rutherford (2013) for more details on the calibration of these models.

Table 8: Model equations

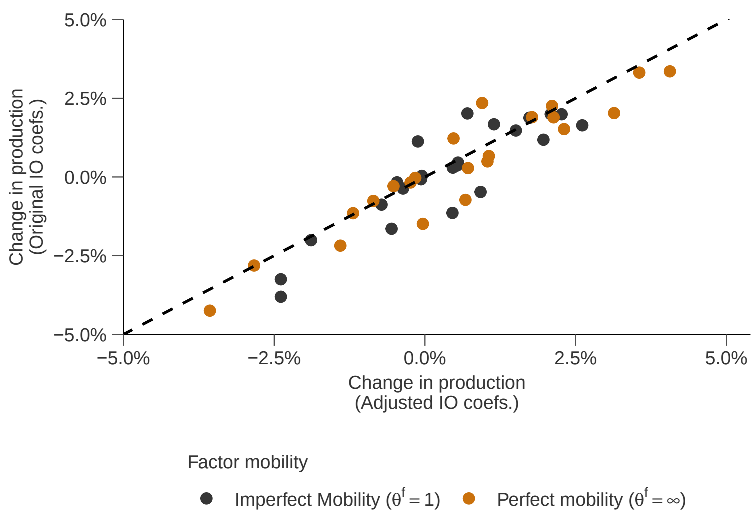
Equation	Associated Variable
Partial Equilibrium Model	
Expenditure on domestic goods (1)	V_{jj}^g
Expenditure on imported goods (2)	$V_{F_j}^g$
Bilateral expenditure (3)	V_{ij}^g
Demand for the average firm (4)	$c_{ij}^g(\tilde{\varphi})$
Composite good price index (5)	P_j^g
Imports price index (6)	$P_{F_j}^g$
Bilateral price index (7)	P_{ij}^g
Input bundle supply (78)	Y_i^g
Optimal price (11)	$p_{ij}^{gt}(\tilde{\varphi})$
Bilateral average firm productivity (17)	$\tilde{\varphi}_{ij}^g$
Zero cuttoff productivity condition (16)	N_{ij}^g
Free entry condition (19)	M_i^g
Total expenditure (79)	V_i^g
Input bundles market equilibrium (27)	z_i^g
General Equilibrium Model	
Expenditure on domestic goods (1)	V_{jj}^g
Expenditure on imported goods (2)	$V_{F_j}^g$
Bilateral expenditure (3)	V_{ij}^g
Composite good price index (5)	P_j^g
Imports price index (6)	$P_{F_j}^g$
Bilateral price index (80)	P_{ij}^g
Unit cost of the input bundle (8)	Y_i^g
Income (21)	I_i
Domestic absorption (22)	$V_i^{g,hh}$
Intermediate expenditure (23)	$V_i^{g,int}$
Total expenditure (24)	V_i^g
Production factor supply (25)	l_i^{gf}
Input bundle market equilibrium (81)	z_i^g
Production factor market equilibrium (28)	w_i^{gf}

Table 9: Armington elasticities for Brazil used in simulations

Sector	Ours			GTAP			CFRT		
	σ^g	ω^g	ζ^g	σ^g	ω^g	ζ^g	σ^g	ω^g	ζ^g
Crop and animal production	6.51	1.80	2.17	3.52	1.76	1.75	5.8	2.90	1.75
Forestry and logging	6.51	1.80	1.79	2.86	1.43	1.75	5.8	2.90	1.75
Fishing and aquaculture	6.51	1.80	1.89	1.43	0.71	1.75	5.8	2.90	1.75
Mining and quarrying	2.53	2.24	1.48	7.53	3.76	1.75	8.3	4.15	1.75
Food products, beverages and tobacco	3.94	2.24	1.53	3.38	1.69	1.75	3.7	1.85	1.75
Textiles, wearing apparel and leather products	4.44	2.52	1.61	4.38	2.19	1.75	3.7	1.85	1.75
Wood products	3.72	2.37	1.71	3.89	1.94	1.75	3.7	1.85	1.75
Paper and paper products	4.01	2.17	1.51	3.37	1.69	1.75	3.7	1.85	1.75
Printing and reproduction of recorded media	3.26	2.24	1.92	3.37	1.69	1.75	3.7	1.85	1.75
Coke and refined petroleum products	2.50	2.24	1.31	2.40	1.20	1.75	3.7	1.85	1.75
Chemicals and chemical products	2.68	1.85	1.53	3.77	1.89	1.75	3.7	1.85	1.75
Pharamaceuticals and medical chemical products	4.05	2.89	1.35	3.77	1.89	1.75	3.7	1.85	1.75
Rubber and plastic products	3.29	2.27	1.53	3.77	1.89	1.75	3.7	1.85	1.75
Non-metallic mineral products	2.53	2.24	1.68	3.31	1.66	1.75	3.7	1.85	1.75
Basic metals	3.42	2.04	1.49	4.09	2.04	1.75	3.7	1.85	1.75
Fabricated metal products	3.13	2.64	1.76	4.29	2.14	1.75	3.7	1.85	1.75
Computer and electronic products	5.69	2.24	1.54	5.03	2.51	1.75	3.7	1.85	1.75
Electrical equipment	3.13	2.73	1.53	5.03	2.51	1.75	3.7	1.85	1.75
Machinery and equipment (n.e.c.)	3.13	1.88	1.58	4.63	2.31	1.75	3.7	1.85	1.75
Motor vehicles, trailers and semi-trailers	4.68	3.63	1.50	3.20	1.60	1.75	3.7	1.85	1.75
Other transport equipment	3.26	2.24	1.53	4.91	2.46	1.75	3.7	1.85	1.75
Other manufacturing	3.43	2.24	1.77	4.29	2.14	1.75	3.7	1.85	1.75
Repair and installation of machinery and equipment	3.36	2.24	1.85	4.29	2.14	1.75	3.7	1.85	1.75
Services	3.26	2.24	1.80	2.33	1.16	1.75	2.8	1.40	1.75

Note: "Ours" represents the elasticities presented in Tables 2 and 3. For some sectors, averages of some estimates were considered for compatibility between the estimation sectors and the WIOD sectors. In the other columns, GTAP and CFRT (Caliendo et al. (2023)) elasticities are presented considering the compatibility with the WIOD sectors.

Figure 4: Impact of input-output coefficient adjustment on sectoral production results.



Note: In this analysis, simulations assuming perfect competition in all sectors were used and based on the Armington elasticities obtained in this study. The 45-degree line represents a reference for evaluating the degree of change in simulation results.

Table 10: Change in production for different sets of elasticities.

	Imperfect Mobility ($\theta^f = 1$)			Perfect Mobility ($\theta^f = \infty$)		
Sector	Ours	GTAP	CFRT	Ours	GTAP	CFRT
Crop and Animal Production	2.00	1.20	1.46	3.09	1.25	1.59
Forestry and logging	0.40	0.19	0.31	0.31	-0.23	-0.18
Fishing and aquaculture	0.47	0.02	0.43	0.44	-0.38	0.02
Mining and quarrying	3.21	3.89	3.53	7.84	12.42	11.38
Food Products, Beverages and Tobacco	3.32	2.89	2.78	4.09	2.83	2.65
Textiles, Wearing Apparel and Leather Products	-5.12	-3.70	-2.61	-8.34	-6.49	-4.46
Wood Products	2.78	2.70	2.39	4.03	3.06	2.58
Paper and Paper products	3.55	3.25	3.03	4.78	3.34	3.00
Printing and Reproduction of Recorded Media	0.04	0.48	0.44	-0.12	0.29	0.22
Coke and Refined Petroleum Products	2.24	1.53	2.12	2.57	1.45	1.98
Chemicals and Chemical products	0.90	1.35	1.12	0.96	0.80	0.55
Pharamaceuticals and Medical Chemical Products	-0.37	0.27	0.13	-1.34	-0.54	-0.74
Rubber and Plastic products	-1.12	0.01	-0.06	-1.64	-0.59	-0.67
Non-metallic Mineral Products	-0.38	0.83	0.65	-0.74	0.57	0.29
Basic Metals	4.02	4.56	3.86	5.30	4.82	3.85
Fabricated Metal Products	-4.03	-1.15	-0.73	-6.81	-2.47	-1.70
Computer and Electronic Products	-0.87	-2.28	-1.20	-1.35	-4.15	-2.16
Electrical Equipment	-6.75	-2.76	-1.33	-11.10	-5.50	-2.70
Machinery and Equipment (n.e.c.)	-0.24	-0.53	-0.20	-0.72	-2.60	-1.52
Motor Vehicles, Trailers and Semi-Trailers	-2.81	0.72	0.26	-4.47	0.31	-0.39
Other Transport Equipment	3.51	4.26	3.27	5.64	5.43	3.54
Other Manufacturing	2.31	2.28	1.86	3.28	2.62	1.98
Repair and Installation of Machinery and Equipment	-	-	-	-	-	-
Services	0.21	0.15	0.18	0.15	-0.02	0.00

Note: The parameter θ_i^f governs factor mobility between sectors. Each simulation was run using a different set of elasticities. 'Ours' means that the elasticities from Tables 2 and 3 were used. In GTAP columns, the GTAP elasticities were considered for all regions. CFRT indicates that the elasticities from Caliendo et al. (2023) were considered for all regions. Welfare is computed as the change in real income (I_i/P_j^{hh}). Real value added is the change in total remuneration of production factors divided by the consumer price index (P_j^{hh}).